This paper provides a detailed summary of the changes made to the spatial measurement content strand from previous editions of *Everyday Mathematics* to *Everyday Mathematics 4*. The recommendations for changes were developed after a careful review of the current research about how children learn mathematics and, more specifically, how they learn about spatial measurement. In addition, these recommendations were influenced by many conversations with Jack Smith, Michigan State University. To read a summary of the research that informed these changes, please see the paper entitled "*Everyday Mathematics 4* Review of Literature for Measurement Strand".

Please note that these recommendations were made prior to the commencement of the formal writing process of *Everyday Mathematics 4*. As Grade Level Leaders worked to enact these, and other recommendations, they often found that they needed to make changes based on what they found as they wrote and field-tested lessons. As a result, you may notice slight differences between these recommendations and the actual content of *Everyday Mathematics 4*. It is reasonable to assume that these differences are the results of decisions made during the formal writing and field-testing process, and were enacted after consulting with the authors of these recommendations. For more information about the writing process or the field testing work for *Everyday Mathematics 4*, please see "*Everyday Mathematics* and the Field-Testing Process."

Broad-Strokes Changes to the Spatial Measurement Strand in *Everyday Mathematics* 4

The recommendations listed in bold below are the broad recommendations that were made for changes across the K-6 curriculum for measurement. Each recommendation is followed by a brief justification for the recommendation.

Incorporate conceptual understandings related to measurement, such as unit iteration and the additive properties of measures in the language that describes what students should know and are able to do.

The research-learning trajectory emphasizes the importance of the conceptual underpinnings of measurement, particularly understanding of units and unit iteration.

Ensure a strong treatment of unit iteration within our trajectory in all dimensions.

At the highest level, the materials need to:

- Do more to help students make the connection between length-unit iteration and the pre-iterated units on rulers and other measuring tools (e.g., physically moving units, laying units end to end to "build" a ruler, etc.).
- Do more to help students make the connection between area and volume/capacity-unit iteration and other ways of finding area (counting squares or cubes, using formulas) (e.g., physically moving and/or accumulating tiles and cubes).
- Delay the introduction of standard measuring tools for length and de-emphasize (at least at the earliest grades) the procedural "tricks" of lining up with the zero mark, reading the number, etc.
- Do more with broken rulers, lining objects up at something other than the zero mark, displacing objects and measuring tools, etc.

- Be more explicit about the connection between iterating length units and iterating area and volume/capacity units (e.g., make explicit connections between iterating and tiling and building with cubes).

Research from the University of Chicago's Spatial Intelligence and Learning Center (SILC), Jack Smith, and others strongly indicates that students need to connect unit iteration to measuring tools in order to understand the tools and procedures for using them. Research also indicates that if students experience a heavily procedural approach to measurement early (especially if standard measuring tools are not connected to a well-established understanding of units and unit iteration), they often have difficulty with broken-ruler problems and other similar measurement situations and challenges later, even after they appear to be proficient measurers.

Include incorrect tools and procedures as a pedagogical strategy and capitalize on opportunities to discuss them.

Research from Jack Smith, SILC, and others suggest that the use of incorrect tools and procedures will help students better understand measurement tools and techniques. More generally, we know that strategic pedagogical use of non-examples can be effective.

Ensure strong connections among length, area, and volume/capacity in the following ways (as well as others):

- Use manipulatives that support measurement of length, area, and volume/capacity, and discuss which dimension of the object you are using to measure: length vs. area vs. volume/capacity. (Key idea: objects are not units; an object carries multiple units.)
- For standard units, use square-inch pattern blocks and centimeter cubes as manipulatives.

Jack Smith's research indicates that most curricula (including EM) have not historically helped students make these connections; he further suggests that students' understanding will likely be enhanced and their learning may be more efficient if they have opportunities to connect how measuring (and manipulating) length works to how measuring (and manipulating) area and volume work.

Jack Smith also indicates that using manipulatives such as square-inch pattern blocks and centimeter cubes to measure multiple dimensions is practical, and it also provides valuable opportunities for students to discuss which dimension they need to pay attention to as they measure, thereby strengthening their understanding of length vs. area vs. volume.

Include clear trajectories for learning about area and volume.

There is considerable research that suggests a progression for area learning that includes, among other things, understanding area as "covering," covering surfaces with various units, counting unit squares, and considering why squares are a good unit for measuring area, and using those activities to understand area formulas. Jack Smith indicates that most curricula (including EM) have not done a thorough job with these concepts in the past.

Incorporate varied opportunities for things to measure, including the following:

- For length, students should measure things that are not straight, or what we might call "paths." (Currently, we tend to focus on line segments on a page and straight edges of objects.) We should connect perimeter activities to path measurement.
- In all dimensions (length, area, and volume/capacity), students should measure more real objects from their everyday experiences, as well as composite and complex shapes. This supports a few goals:
  - Emphasizing the "everyday" in *Everyday Mathematics*
  - Dispelling the notion that area is only an attribute of rectangles
  - Supporting the additive properties of area and volume/capacity
  - Supporting the idea that you can (often must) measure length with more tools than just a ruler (e.g., string, tape measures)

Nguyen's[1] research suggests that thinking of length along a path helps students develop concepts of additivity, conservation, and transitivity of length. Other researchers also identify measuring paths, in addition to simply measuring segments and sides of objects, as valuable.

Jack Smith suggests that incorporating a wider variety of objects will also help students sort out the dimensions with which they are working, help them develop additivity concepts and decomposition/composition skills, and ensure that measurement skills are being consistently transferred/applied to real-world situations.

Across all dimensions, say that "all measures are approximations," instead of saying that "all measures are estimates."

In *Everyday Mathematics* Third Edition, we told students that all measures are estimates, but also asked them to estimate measures. These are two slightly different uses of the word *estimate* that are potentially confusing. In *Everyday Mathematics 4,* we propose using the terminology "all measures are approximations," which allows us to continue to ask students to estimate measures without confounding the language. This should be consistent across measurement, including time, mass, temperature, etc.

Consistently apply the terms "precision" and "accuracy" across all grades.

In *Everyday Mathematics* Common Core Edition, we began to use the word "precision" to refer to the refinement of units that are being used, and "accuracy" to refer to how close a measurement is to the actual measure. Ensuring that these terms are used consistently will also make the language more mathematically sound.

Capitalize on some of the important measurement practice and application opportunities that exist in other strands.

Battista's research shows that number and counting help lead to quantifying space, that composing/decomposing is integral in comparing amounts of space, and numerical operations apply to measures of space. These skills are taught through the Number and Numeration, Geometry, and Operations strands as well as being important in Measurement.

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[1] See the "*Everyday Mathematics 4* Review of Literature for Measurement Strand" paper for the full citations.