This paper provides a detailed summary of the changes made to the facts content strand from previous editions of *Everyday Mathematics* to *Everyday Mathematics 4*. The recommendations for changes were developed after a careful review of the current research about how children learn mathematics and, more specifically, how they learn facts. To read a summary of the research that informed these changes, please see the paper entitled "*Everyday Mathematics 4* Review of Literature for Facts Strand".

Please note that these recommendations were made prior to the commencement of the formal writing process of *Everyday Mathematics 4*. As Grade Level Leaders worked to enact these and other recommendations, they often found that they needed to make changes based on what they found as they wrote and field-tested lessons. As a result, you may notice slight differences between these recommendations and the actual content of *Everyday Mathematics 4*. It is reasonable to assume that these differences are the results of decisions made during the formal writing and field-testing process, and were enacted after consulting with the authors of these recommendations. For more information about the writing process or the field-testing work for *Everyday Mathematics 4*, please see "*Everyday Mathematics* and the Field-Testing Process."

Broad-Stroke Changes to the Facts Strand in EM4

Addition and Subtraction Facts:

Engage students in conceptual strategy development for addition facts using ten frames.

Research has documented the effectiveness of using quick images with ten frames to encourage subitization, to help students find ways to decompose numbers flexibly, and to develop addition fact strategies such as making ten (Kline, 1998; Hollister, 2009; Kling, 2011; Wheatley and Reynolds, 1992; Clements, 1999). An important point about strategy development is that "direct teaching of these strategies must be done conceptually rather than simply by using imitation and repetition" (NRC 2001, p. 188). Consistent with this, in her 1985 study of derived fact strategies, Ruth Steinberg found that when new derived fact strategies were taught in class, some students would treat them as rote procedural rules without understanding the relationships among the number facts (Steinberg, 1985). This shows that for true fluency to be achieved, students must have an opportunity to develop these strategies for *themselves* with meaning. Carol Thornton (1978) found that many children seemed to adopt strategies that were explicitly taught, encouraged, or otherwise suggested during instruction. As students develop strategies, they must be made explicit because not all children will invent strategies for addition and subtraction facts (Thornton, 1978). One technique to nudge students toward particular strategies while still promoting student ownership is to focus whole class discussion around slightly more advanced procedures that some students are using. Ten frames and double ten frames provide a natural vehicle for doing so.

The Common Core State Standards clearly focus on determining the complement to ten in Kindergarten (K.OA.4). Ten frames provide an excellent structure for encouraging

children to do so, more so than other tools such as cube trains (Losq, 2005). Research (Wheatley and Reynolds, 1992; Kling, 2011; Hollister 2009; Kline, 1998; Clements, 1999) has suggested that quick images with ten frames will allow students to move towards subitization of numbers, find ways to decompose numbers (K.OA.3), and motivates strategy development such as making ten (1.OA.6, 2.OA.2).

Concentrate specifically on the combinations of ten and doubles in Kindergarten and Grade 1 by having students engage in meaningful practice with those facts, with a goal of having students achieve automaticity with those facts by the end of first grade.

Baroody (Baroody, 1985) and Carpenter and Moser (Carpenter & Moser, 1984) have established that it is not realistic to expect that all families of basic addition or subtraction combinations can be mastered in one year's time. Thus it makes sense to begin focusing on the most critical groups of facts in Kindergarten and Grade 1 if we hope students will have all addition facts automatic by the end of Grade 2. The importance of knowing combinations of ten is well-documented (Fuson & Kwan, 1992; NRC, 2001; Henry and Brown, 2008), as is the correlation between knowing these facts and the ability to apply the making-ten strategy fluently (Fuson & Kwan, 1992; Steinberg, 1985). As making ten is one of the key strategies for finding addition facts as well as subtraction facts (Thornton, 1990), we suggest that a priority on developing automaticity with these will help students develop fluency with related facts within Grade 1 and automaticity with related facts by the end of Grade 2. Similarly, doubles can be used to fluently derive the near doubles (facts 1 or 2 away from a double), which includes many of the facts that tend to be challenging for students, as well as "subtraction doubles" such as 12 – 6 (Thornton, 1990). The ease of memorizing doubles is well-documented throughout research (NRC, 2001; Thornton, 1978; Steinberg, 1985; Henry & Brown, 2008); thus, focusing on helping children develop automaticity with these facts within Grade 1 is both realistic and useful for developing fluency with all facts. Furthermore, subtraction fact strategies that will be encouraged in EM4 make use of the combinations of ten and the doubles, and automaticity with these facts will promote fluency with subtraction facts as well (Thornton, 1990).

Combinations of ten and/or making ten are explicitly referenced in the Common Core State Standards in Kindergarten and Grade 1 (K.OA.4, 1.OA.6), and implicit in Grade 2 (2.OA.2). Furthermore, as key facts that can be used to efficiently derive many other facts, learning these is important for supporting automaticity with all addition facts and fluency with subtraction facts (2.OA.2).

Provide targeted practice of facts throughout Grades K–2 to encourage strategy development and eventual fluency.

According to findings summarized in *Adding It Up* (NRC, 2001), both targeted and cumulative practice is needed. They recommend "targeted forms of practice, with particular combinations that have yet to be mastered or on which efficient procedures can be used" as opposed to a "scattershot" form of practice. Furthermore, according to Baroody, "Practice should focus on making reasoning strategies more automatic, not on

drilling isolated facts" and "the learning and practice of number combinations should be done purposefully" (Baroody, 2006 p. 201). Interview data from Thornton's study (Thornton, 1978) strongly indicated that many children seemed to adopt strategies that were explicitly taught, encouraged, or otherwise suggested during instruction. This suggests that practice focused around particular groups of facts that utilize a particular strategy is likely to encourage students to develop mastery with that and related strategies (e.g., practicing combinations of ten will encourage mastery of those facts as well as facilitate use of the making ten strategy).

There is explicit focus in the CCSS on strategy development and use with regards to fact work (1.OA.6, 2.OA.2). Thus, it makes sense to provide increased, meaningful practice of these strategies to ensure mastery of them.

Sequence subtraction fact work in Grades 1 and 2 to focus on strategies, with some delay from addition facts.

Thornton (Thornton, 1990) conducted two sequential Grade 1 studies that experimented with a strategy-based teaching sequence for introductory subtraction facts. Both years of the study showed much greater success for the students using the experimental approach to subtraction facts. This work suggests that a logical sequence for focusing on subtraction facts in Grade 1 is (1) "count backs" (– 1, – 2, – 3 facts); (2) subtraction doubles; (3) "ten-frame facts" (10 – 9, 10 – 8, 10 – 7, 10 – 6, 10 – 4, 9 – 5, 9 – 4); (4) "count ups"; and finally (5) harder facts. To follow-up on this, Steinberg (Steinberg, 1985) utilized the following sequence for subtraction facts for second graders: (1) fact families; (2) subtraction or "reverse doubles"; (3) reverse near doubles; (4) going through 10. These sequences are reflected in the relevant sections of the specific grade level recommendations.

Thornton also found that encouraging students to "Add to Check" once they developed some fluency with a new strategy helped encourage recognition of the relationship between addition and subtraction and increased use of known addition facts to solve for unknown subtraction facts. She recommends this as a precursor to work with fact families as well. Furthermore, Thornton recommends delaying the introduction to subtraction facts until after a number of addition facts have become automatic, to increase the likelihood of students using the addition/subtraction relationship to solve subtraction facts. In her 1990 study, she utilized a 2-week delay between early addition and subtraction units during which unrelated mathematics topics and frequent small amounts of addition facts practice was used, an approach grounded in earlier research (Steinberg, 1985; Thornton, 1984). Her work as well as other studies (Steinberg, 1985; Henry and Brown, 2008; NRC, 2001) have suggested that subtraction facts are more challenging than addition facts for children. This research supports delaying work with subtraction facts and expecting less fluency with those facts than addition facts in Grade 1.

The CCSS only expects fluency with subtraction facts within 10 by the end of Grade 1 (1.OA.6), and never expects automaticity with subtraction facts (2.OA.2). Thus it is reasonable to follow the recommendations from research to have subtraction-fact work

follow sufficient addition-fact work. Furthermore, the CCSS explicitly identifies several of the strategies related to our focus for addition and subtraction facts, including using the relationship between addition and subtraction (1.OA.4, 1.OA.6) and decomposing a number leading to a ten (1.OA.6).

Multiplication and Division Facts:

Introduce multiplication early in Grade 3. Include a variety of interpretations and strategies.

Clark and Kamii (Clark & Kamii, 1996) found that formal multiplicative thinking appears early (among 45% of second graders in their study) but develops very slowly (only 49% of fifth graders in their study were found to be solid multiplicative thinkers). They found that some children struggled to think multiplicatively, and instead continued to think additively. Similarly, Anghileri (Anghileri 1989) found that the majority of children in her study (75%) used at least three different approaches to multiplication on a single assessment; thus, these levels of multiplicative thinking are very fluid. She found that even many older children tended to use number patterns rather than recall of multiplication facts. All of this suggests the need to carefully bridge students' thinking between earlier approaches such as modeling, skip counting, and repeated addition to more formal multiplicative thinking. (2.OA.3, 2.OA.4, 2.NBT.2) Given the CCSS demands with regards to multiplication in Grade 3, this research suggests it is imperative that multiplication work begin as soon as possible so that sufficient conceptual groundwork can be laid before intense fact work begins.

Be deliberate about multiplication strategy development. Provide sufficient time for this development.

"Practice that follows substantial initial experiences that support understanding and emphasize 'thinking strategies' has been shown to improve student achievement with single-digit calculations. This approach allows computation and understanding to develop together and facilitate each other" (NRC 2001, p. 193). Rathmell (Rathmell, 1979) argues for allowing sufficient time for strategies to be understood. Typically, ample time is not given to the development and meaningful practice of key strategies. Steinberg (Steinberg, 1985) found there was a delay of two to four lessons between the time a new strategy was taught in class and the time most children started using it by choice. This occurred despite the fact that most of the children could successfully use the new strategy in the first or second lesson when asked by the teacher. Thus it is important that strategies are not presented in rapid succession. Thornton (Thornton, 1990) further supports this, as in her study she would make a new strategy the only major focus for that particular lesson, and follow in subsequent lessons with short, frequent practice of facts.

Strategy use for determining facts is explicit in the CCSS (3.OA.7), so this focus on providing sufficient time for strategy development is consistent with that goal.

Ensure careful timing and sequencing of the introduction of particular multiplication facts.

Research agrees on the importance of introducing multiplication facts in terms of their relative difficulty, not in terms of factor size as in the traditional progression (Thornton, 1978; NRC, 2001; Heege, 1985; Kamii and Anderson, 2003). Dossey and Cook (Cook & Dossey, 1982) found more rapid growth in fact mastery when a "thinking strategies" approach is used, and Heege (1985) similarly argues that children become so skilled applying their informal thinking strategies that they eventually memorize their facts. Thus focusing the learning of facts around common strategies, not by factor size, is preferable. The specific sequence recommended (see below) is based on a combination of the work of Thornton (Thornton, 1978), Heege (Heege, 1985), and Anderson and Kamii (Anderson & Kamii, 2003), and Flowers and Rubenstein (Flowers & Rubenstein, 2010).

Given the challenge of 3.OA.7, which requires automaticity with multiplication facts and fluency with division facts by the end of Grade 3, careful sequencing of fact work to best ensure the development of fluent strategies is essential. Furthermore, the CCSS for Grade 3 include the expectation that children apply the Commutative, Associative, and Distributive Properties as strategies to multiply and divide, although they needn't explicitly name the properties (3.OA.5). The recommendations make use of these properties to help students carefully develop meaningful and effective strategies.

Utilize targeted practice of facts to facilitate strategy development.

The research on targeted practice of addition and subtraction facts also applies to multiplication facts.

While specific multiplication fact strategies are not identified in the CCSS, the use of properties of operations is recommended for developing fluency (3.OA.5, 3.OA.7). In particular, the Distributive Property would be effectively promoted by careful selection of fact practice and supports various strategies involving decomposing in order to relate to a known fact (finding 3s by relating to 2s, finding 7s by relating to 5s, finding many "near squares", etc.).

Develop fluency with division facts by relating them to the corresponding multiplication facts with minimal delay.

In EM, the approach has always been to build strong connections between the inverse operations, particularly multiplication and division. This recommendation will just be making a foundational idea of EM a more explicit strategy within the program.

Division can be introduced at the same time by relating it to multiplication. Research suggests that children determine the operation (multiplication or division) by the position of the unknown (Kouba, 1989). Thus, if a factor is unknown, children will utilize division, often even if it has not been explicitly taught (Mulligan and Mitchelmore, 1997). According to Mulligan and Mitchelmore (Mitchelmore, 1997), children

spontaneously relate multiplication and division and do not necessarily find division more difficult. Furthermore, they found throughout their study remarkable improvement on division scores even though division was not explicitly taught. Thornton (1978) also focused on having children "think of the multiplication fact" in her study, leading to improved success on division.

The CCSS clearly recommends using the relationship between multiplication and division to determine division facts (3.OA.6, 3.OA.7). Furthermore, fluency with all division facts is expected by the end of Grade 3, and given the related expectation of automaticity with all multiplication facts within the same time frame, it makes sense to utilize the relationship between the operations to reduce the cognitive load of learning so many facts.

Specific Facts Recommendations by Grade Level

Addition and Subtraction Facts

Kindergarten

The main change to the Kindergarten activities involves altering and expanding the use of ten frames and quick images in the curriculum. In a broad sense, this would include (1) allowing children more freedom in how they fill their ten frames so that a greater variety of representations for numbers will be produced, including some which will promote particular strategies that wouldn't emerge without this freedom; and (2) using quick images with dot patterns, five frames, and ten frames to encourage subitization, flexible decomposition, and strategy development. These changes will expose students to new ways of using these representations and provide children with more opportunities to develop fluency within five, subitize, learn to decompose/compose numbers in flexible ways, and develop strategies for finding addition combinations. More specifically, these recommendations will address CCSS K.OA.3, K.OA.4, and K.OA.5 in meaningful and engaging ways.

Grade 1

Once again, several of the main changes to the Grade 1 lessons involve altering and/or including the use of ten frames and quick images in the curriculum. In a broad sense, this will include (1) allowing children more freedom in how they fill their ten frames so that a greater variety of representations for numbers will be produced, including some which will promote particular strategies that wouldn't emerge as easily without this freedom; (2) using quick images with ten frames to encourage subitization and strategy development; and (3) incorporating some quick images with dot patterns as well, as another form of meaningful practice.

Another main focus for revision will be on the ordering of how facts are introduced, with a heavy focus on doubles and combinations of ten beginning in Unit 4, followed later in

the year by the near doubles and making-ten strategies. The language of "shortcuts" should be removed as this implies a trick instead of a strategy. All of these changes will expose children to new ways of using representations and provide them with more opportunities to subitize, learn to decompose and compose numbers in flexible ways, develop fluency within ten, develop automaticity with doubles and combinations of ten, and develop strategies for finding other addition combinations. More specifically, these recommendations will address CCSS 1.OA.3 and 1.OA.6 in meaningful and engaging ways.

Grade 2

One of the main changes to the Grade 2 lessons is including the use of ten frames and quick images with ten frames in the curriculum, particularly to encourage strategy development and to provide meaningful practice with addition facts. The language of "shortcuts" should be removed as this implies a "trick" instead of a strategy. Another main focus for revision will be on reviewing the key groups of facts of combinations of ten and doubles early on, so these facts are more readily available to children in subsequent strategy development. Maintaining these facts and practicing near doubles and making-ten strategies will be a priority in this grade, as will be meaningful practice of these facts, with the hope that this will prepare children to meet CCSS 2.OA.2.

Grade 3

Given the CCSS and GLG for automaticity with addition facts and fluency with subtraction facts by the end of second grade, the work in Grade 3 is largely focused on fact maintenance. This maintenance, although not explicitly required in the CCSS, is recommended in order to facilitate multidigit computation work and meeting CCSS expectations such as 3.NBT.2. There is only one lesson in Grade 3, namely Lesson 2-1, which includes review of addition and subtraction facts. All other practice will be in the Ongoing Learning and Practice sections or MMFs.

Multiplication and Division Facts

Grade 2

Most of the multiplication and division work in Grade 2 is very conceptual in nature, focusing on ideas such as equal groups, array models, and solving number stories.

Grade 3

The recommendations for multiplication facts in Grade 3 will require a number of substantial changes. First of all, given the CCSS expectation that third graders develop automaticity with their multiplication facts by the end of third grade (3.OA.7), it is necessary to begin fact work as early as possible. Conceptual review, particularly making use of the Grade 2 facts of 2s, 5s, and 10s, will begin in Unit 1. Secondly, the

content of the lessons will be refocused so that facts are encountered based on the sequence identified by the facts working group, which follows a trajectory supported by research and focusing on strategy development. In this trajectory, the Grade 2 focus facts of 2s, 5s, 10s, and 1s will be followed in Grade 3 by 0s, squares, 9s, 3s. Lastly, all other facts will be derived through decomposition, relating to squares or tens, or doubling.

Division facts will be addressed largely by relating to multiplication through fact families, with minimal delay between the introduction of a new group of multiplication facts and practice incorporating the related division facts. Finally, the nature of practice, particularly with routines such as *Beat the Calculator* and Fact Triangles, will change to reflect the greater emphasis on strategy development. These changes are outlined in further detail below.

Grade 4

Given the CCSS and GLG for automaticity with multiplication facts and fluency with division facts by the end of third grade, the work in Grade 4 is largely focused on fact maintenance. This maintenance, although not explicitly required in the CCSS, is recommended in order to facilitate multi-digit computation work and meeting CCSS expectations such as 4.OA.4 and 4.NBT.5.

Fact Strategies in EM4

✓ = Instruction **P** = Practice and Application

	Fact Strategies	**K**	**1**	**2**	**3**
Addition	Sums within 5 (1 + 4, 2 + 2, etc.)	✓	**P**		
	+ 0, 1, or 2	✓	**P**		
	Doubles (2 + 2, 6 + 6, etc.)		✓	**P**	
	Combinations of 10 (3 + 7, 8 + 2, etc.)	✓	✓	**P**	
	Near Doubles (6 + 7, 8 + 7, etc.)		✓	**P**	
	Making Ten (8 + 3, 9 + 5, etc.)		✓	**P**	
Subtraction	**Fact Strategies**	**K**	**1**	**2**	**3**
	Think Addition		✓	**P**	
	Counting Up		✓	**P**	
	Counting Back	✓	✓	**P**	
	– 0 and – 1	✓	✓	**P**	
	Using Doubles		✓	**P**	
	Going Back Through 10			✓	
	Going up Through 10			✓	

	Fact Strategies	**2**	**3**	**4**	**5**
Multiplication	2s, 5s, and 10s	✓	✓	**P**	
	0s, 1s, Multiplication squares (2 x 2, 3 x 3, etc.)		✓	**P**	
	Adding a Group		✓	**P**	
	Subtracting a Group		✓	**P**	
	Halving and Doubling		✓	**P**	
	Near Squares (adding or subtracting a group to a square)		✓	**P**	
	Breaking Apart		✓	**P**	
	Extended Facts		✓	✓	**P**
Division	**Fact Strategies**	**2**	**3**	**4**	**5**
	Think Multiplication (use related multiplication fact)		✓	**P**	
	Extended Facts			✓	**P**

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