This paper provides a detailed summary of the changes made to the operation content strand from the Third Edition of *Everyday Mathematics* and CCSS edition to *Everyday Mathematics 4*. The recommendations for changes were developed after a careful review of the current research about how children learn mathematics and, more specifically, how they learn about operations. To read a summary of the research that informed these changes, please see the paper entitled "*EM4 Review of Literature for Operations Strand*".

Please note that these recommendations were made prior to the commencement of the formal writing process of *Everyday Mathematics 4*. As Grade Level Leaders worked to enact these, and other recommendations, they often found that they needed to make changes based on what they found as they wrote and field-tested lessons. As a result, you may notice slight differences between these recommendations and the actual content of *Everyday Mathematics 4*. It is reasonable to assume that these differences are the results of decisions made during the formal writing and field-testing process, and were enacted after consulting with the authors of these recommendations. For more information about the writing process or the field testing work for *Everyday Mathematics 4*, please see the "*Everyday Mathematics* and the Field-Testing Process."

## Summary of the Operations Strand in *Everyday Mathematics* 4

The recommendations listed in bold below are the broad recommendations that were made for changes across the K-6 curriculum for operations. Each recommendation is followed by a brief justification for the recommendation.

***Include work with flexible representations of numbers in Kindergarten through Grade 2, with emphasis on understanding 1 ten as both a constructed, countable unit and as a group of 10 ones (and, by extension, understanding 1 hundred as both a constructed, countable unit and as a group of 10 tens or 100 ones).***

The research strongly indicates that success with computation is dependent on a flexible understanding of number, including being able to flexibly switch back and fourth from thinking of ten as 1 ten to thinking of a ten as 10 ones (Fostnot and Dolk 2001b; Clements and Sarama 2009).[1]

***Make strong connections between representations (e.g. base-10 blocks and expanded notation) and computation methods in Grades 2-4.***

The research strongly indicates that students have more success with computation when algorithms and methods are taught with understanding (NRC 2001). Clearly connecting representations such as base-10 blocks and expanded notation to computation methods is likely to make the underlying concepts more salient to students.

***Present open number lines as a tool for thinking.***

Research suggests that the open number line can be a powerful aid for thinking and communicating strategies when it is treated as a flexible tool (Fosnot and Dolk 2001b; Verschaffel, Greer, and De Courte 2007).

***Include work with area models.***

The research suggests that use of area models helps students to make sense of partial products, to make sure they have found all of the partial products, and to connect strategies for multiplying larger number to work they have already done with multiplying smaller numbers (Fosnot and Dolk 2001a; NRC 2001).

***Provide opportunities for children to compare, connect, and discuss the efficiency of various strategies.***

The basic building blocks for this work are already present in EM. This recommendation involves making a foundational idea of EM more explicit and consistent within the program.
The research clearly indicates that the process of analyzing and comparing methods supports understanding of computation (NRC 2001). Comments from the external reviewer confirm that our proposed changes reflect current research findings.

***Introduce the U.S. traditional algorithms.***

Alternative algorithms have always been a part of EM. This recommendation aims to keep these long-standing features in the program while still keeping the overarching operations themes of connecting place value to computation and not introducing too many algorithms at once.
These changes will ensure that both the alternative and standard algorithms are introduced at times when they will be both useful and comprehensible for students.
The research indicates that when the standard algorithms are introduced too early, before foundational ideas of place value and solid understanding of the operations are in place, students are not able to use them effectively or flexibly (Kamii and Dominick 1998; NRC 2001).

***Require children to consistently use estimates to judge the reasonableness of their answers.***

EM has always emphasized the importance of estimation, both as a problem-solving strategy in its own right as well as a method of checking that answers are reasonable. These changes simply make these ideas more explicit.

***Emphasize the use of tools such as number lines to aid with rounding before introducing procedures and shortcuts. Connect work with rounding to computational estimation as appropriate.***

These changes will more clearly align EM with the CCSS's approach to rounding and using rounding as an estimation strategy. In the current edition, EM's work with rounding is scattered, disconnected, and sometimes placed in lessons that have a focus that is only tangentially related to rounding. These changes will smooth out the trajectory.

**Summary of Grade-by-Grade Changes to the Operations Strand in EM*4***

The following table provides more detail about the specific recommendations for the operations strand at each grade level, and is followed by several pages that give a broad summary of the changes to the operations content at each grade level.

## Table 1: Strategies & Procedures in EM4

✓ = Instruction **P** = practice
Yellow = new to grade from Third edition of EM

| Operation | | Grade 1 | Grade 2 | Grade 3 | Grade 4 | Grade 5 | Grade 6 |
|---|---|---|---|---|---|---|---|
| **Addition** | Invented strategies | ✓ | ✓ | | | | |
| | Counting Up | | ✓ | ✓ | | | |
| | Partial sums | | ✓ | **P** | **P** | | |
| | Column addition | | | ✓ | **P** | | |
| | U.S. traditional addition | | | | ✓ | | **P** |
| | Application of algorithms to decimals | | | | | ✓ | ✓ |
| **Subtraction** | Invented strategies | ✓ | ✓ | P | | | |
| | Expand-and-trade | | ✓ | **P** | | | |
| | Trade first | | | ✓ | **P** | | |
| | U.S. traditional subtraction | | | | ✓ | | **P** |
| | Application of algorithms to decimals | | | | | ✓ | ✓ |
| **Multiplication** | Invented strategies | | ✓ (facts) | ✓ | | | |
| | Partial products | | | | ✓ | **P** | |
| | Lattice method | | | | ✓ | **P** | |
| | U.S. traditional multiplication | | | | | ✓ | **P** |
| | Application of algorithms to decimals | | | | | ✓ | ✓ |
| **Division** | Invented strategies | | | ✓ | P | | |
| | Partial quotients | | | | ✓ | **P** | **P** |
| | U.S. traditional division | | | | | | ✓ |
| | Application of algorithms to decimals | | | | | ✓ | ✓ |

## Kindergarten

The overall approach to the Kindergarten operations strand remains mostly unchanged in that children begin with and focus on activities that address and develop conceptual understanding of addition and subtraction. As they solve number stories in context, they engage in a range of strategies, progressing from direct modeling strategies, to counting strategies, to beginning to use applied facts strategies; and they continue to represent addition and subtraction with words, objects, pictures, and equations. To lay groundwork for Grade 1 and beyond, Kindergarten children are exposed to a wider range of problem types in number stories, including start-, part-, and change-unknown situations. Increased work with composing and decomposing numbers, with combinations that add to ten, and with place value for numbers 10 through 19 also lends added support to number sense and place-value understanding that will help children understand multidigit computation methods encountered in Grade 1 and beyond.

***Emphasize flexibly composing and decomposing numbers through and including 10.***

***Increase opportunities to discuss ten both as 10 ones and 1 ten.***

Work in these areas will give children stronger number sense and place value understanding that will support understanding of multidigit computation in later grades.

## First Grade

Most of the changes to the first-grade program are related to two topics: discussion and comparison of strategies, and place-value exchanges. For the former, children begin making sense of each other's strategies and compare strategies, continuing to do so as they begin to solve problems with larger numbers toward the middle of the year.

In terms of place-value exchanges, children are given more practice with making exchanges from a larger unit to a smaller unit the goal of helping children understand that place-value exchanges simply create an alternate and equally valid representation of a number. Children are first introduced to these exchanges using base-10 blocks. Soon after, the work is reinforced using money, initially just pennies and dimes, and later, pennies, dimes, and dollars.

***Incorporate more discussion of informal addition and subtraction strategies into lessons.***

***Reduce the focus on representing money amounts using the fewest number of coins, instead emphasizing that there are multiple ways to represent the same money amount.***

***Add discussion around informal addition and subtraction strategies, and encouraged children to compare strategies when appropriate.***

***Increase opportunities to create multiple representations of 2-digit numbers.***

***Continued discussing and comparing informal addition and subtraction strategies.***

**Second Grade**

Children continue to explore place-value exchanges, and begin to record their concrete representations of numbers with number sentences. This place-value work is connected to work with algorithms for addition and subtraction. Children continue to share and compare strategies for addition and subtraction; the focus of Grade 2 will be to compare invented methods with more formal procedures that are introduced in the grade.

Children are encouraged to make connections between the base-10 blocks and the written numbers. Expanded notation will be used to help bridge that gap. Note that this will include adding some expanded notation representations to the lesson on invented subtraction strategies with base-10 blocks.

Children will experience the open number line first as a way of keeping track of counts, then as a way of recording and communicating addition and subtraction strategies. Connections among the number grid, base-10 blocks, the class number line, and the open number line will be explored.

Children begin to make estimates for computation problems in second grade, and they are asked to compare their exact answers to their estimates. This step of using estimates to check the reasonableness of answers is made more explicit.

***Increase opportunities to make exchanges between 1s, 10s, and 100s in both directions (e.g., 10 ones for 1 ten and 1 ten for 10 ones) in and out of the context of money.***

***Continue the practice of comparing invented strategies. As specific addition and subtraction strategies are introduced, give children opportunities to compare them to other strategies.***

***Connect expanded notation to work with using place-value trades to create multiple representations of numbers.***

***Introduce the open number line as a tool for representing strategies for counting, addition, and subtraction.***

***Be more explicit about encouraging children to use estimates to check the reasonableness of answers.***

***Connect the partial-sums algorithm to work with expanded notation. Have children compare the partial-sums algorithm to other addition methods.***

***Connect invented subtraction strategies to work with expanded notation.***

***Connect the expand-and-trade algorithm to the work with expanded notation and place-value trades. Compare the expand-and-trade algorithm to other subtraction methods.***

**Third Grade**

Children will continue to explore place-value trades and connect them to computation methods, including a new algorithm, column addition. Children will expand their use of open number lines to include modeling rounding techniques and illustrating solutions to elapsed-time problems, and they will continue to compare computation strategies. For addition and subtraction, they will compare different algorithms and think carefully about what strategy might be most effective for a given problem. They will also compare and discuss their own invented strategies for multiplication.

***Revise the lesson on number grids to include discussing the number grid as a tool for counting and finding differences. Add a new lesson on the number line that discusses it as a tool for solving problems.***

***Include more discussion of children's strategies for solving number stories.***

- ***Connect place value understanding to rounding and addition and subtraction strategies.***
- ***Connect rounding to estimation techniques, and provide additional practice with using estimates to judge the reasonableness of answers.***
- ***Continue practicing place-value trades.***
- ***Introduce column addition and connect work with expanded notation and trading to this algorithm (as well as to partial sums).***
- ***Allow children to compare partial sums to column addition, and to compare counting up to trade first.***

***Introduce area models in the context of decomposition strategies for facts.***

**Fourth Grade**

As fourth graders continue to learn new methods for computation, they compare them to the methods they already know. They should use their growing place-value understanding to explore and apply the U.S. traditional algorithms for addition and subtraction as well as the lattice algorithm for multiplication and the partial-quotients algorithm for division. We recommend that they continue using open number lines to model and solve addition and subtraction problems, including elapsed-time problems, and that they use area models to make sense of multidigit multiplication and the algorithms associated with it. Making ballpark estimates and using them to judge the reasonableness of answers should

continue to be emphasized. Formal rounding procedures can be introduced and used as an aid in estimation.

***Adjust the work with expanded notation to reflect what children have experienced in previous grades.***

***Introduce the U.S. traditional addition algorithm, and encourage students to compare it to other addition methods they have used.***

***Introduce the U.S. traditional subtraction algorithm, and encourage students to compare it to other subtraction methods they have used.***

- and why. (The computation need not be done every time.)

***Provide more explicit practice with using estimates to judge the reasonableness of answers.***

- ***Introduce formal rounding procedures, and connect rounding to estimation.***
- ***Introduce area models as a way to organize partial products, and also as a way to make connections between the partial-products algorithm and lattice multiplication.***
- ***Continue to use expanded notation to help students connect place-value understanding to computational algorithms.***

***Provide problem contexts and discussions that lead students to division strategies that resemble the partial-quotients algorithm. Use this as an entry point for presenting the algorithm formally.***

**Fifth Grade**

Students continue the practice of comparing computation methods by comparing the U.S. traditional multiplication algorithm to other multiplication methods they know. We further recommend that they continue refining their use of the partial-quotients division algorithm to prepare them for the U.S. traditional long division algorithm in sixth grade. In order to progress toward fluency with decimal addition and subtraction, we recommend that they extend work with rounding and expanded notation to decimals before exploring decimal addition and subtraction. This follows from the previous grades' recommendations for exploring place value before computation with whole numbers.

Fifth graders begin to explore decimal multiplication and division in earnest, to prepare them for fluency with decimal computation with all four operations in sixth grade.

***Introduce the U.S. traditional multiplication algorithm, and compare it to other multiplication methods. Continue to use estimation to judge the reasonableness of answers.***

***Review the partial-quotients method for division, and encourage students to use strategies to find more efficient partial quotients.***

***Address expanded notation and rounding of decimals.***

***Include the U.S. traditional algorithms for decimal addition and subtraction and provide opportunities for students to compare the algorithms.***

***Introduce decimal division in the context of sharing money.***

**Sixth Grade**

The goals for sixth grade in terms of operations are to introduce U.S. traditional long division and to develop fluency with decimal computation with all four operations.. For information about changes to operations with fractions and decimals in Grade 6 see: "Changes to the Fraction Strand in *Everyday Mathematics 4"*.

***Add the names of specific algorithms to the existing lessons on decimal addition, subtraction, and multiplication. Focus on developing estimation skills to place decimal points in multiplication and division done with standard algorithms and using traditional algorithms for addition and subtraction (lining up the decimal point).***

***Introduce the U.S. traditional long division algorithm after the review of the partial-quotients algorithm and compare the algorithms.***

***Mention U.S. traditional long division in the lesson on decimal division.***

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[1] See the "*Everyday Mathematics 4* Review of Literature for Operations Strand" paper for the full citations.