During the early planning phase of *Everyday Mathematics 4*, a team of writers worked together to read and summarize the current body of research about facts. The information that follows is the summary of that research. For more information about the planning phases of *Everyday Mathematics 4*, see the paper "*Everyday Mathematics* and the Writing Process."

Summary of Research

Addition and Subtraction Facts

Children begin developing their concepts of addition and subtraction as early as preschool. Addition is first learned in the context of word problems, and most children entering kindergarten can use counting to solve simple problems set in context (NRC 2001, 183). Carpenter et al. (1993) confirmed the use of direct modeling as the primary method for solving computation problems (435). As they begin solving addition problems, children generally count out objects for both addends and then count all the objects to find the sum. Children typically first understand subtraction as take away (NRC 2001). To solve subtraction problems, they represent the minuend by counting out objects or making drawings, then take away (remove, cross off, etc.) the subtrahend by counting it out, and finally count the difference that remains. It is important to note that single-digit subtraction has been shown to usually be more difficult than addition for children in the United States (NRC 2001, 194).

During the time that children are refining their counting skills and beginning to apply them to solve computation problems set in context, some researchers argue that it is important *not* to allow children to become too reliant on counting. They argue that more sophisticated understandings of the composition of numbers can come from encouraging subitization, or the recognition of small numbers from their representations without needing to count. Wheatley and Reynolds (1999) argue for the necessity of a "collections approach" to number, which allows students to develop a flexible and abstract notion of number. With this flexible notion of number, children can "use the unit in adding or subtracting and can decompose the unit into singleton units" (11). Subitizing helps develop this collections approach. The collections approach is necessary if children are to later develop the number strategies we associate with fluency with addition and subtraction facts. Losq (2005) argues for using ten frames over cube trains of ten for developing this. In particular, quick image activities with ten frames and dot patterns have been shown to help children move beyond counting (Wheatley and Reynolds 1999; Kline 1998).

As children continue working with addition and subtraction situations, they progress from finding sums by modeling and counting all to counting on. This typically occurs in late kindergarten or first grade. Research has shown that counting on is accessible to all first graders and improves speed and accuracy of solutions (NRC 2001, 194). During first grade, children typically progress from using models to aid their counting on procedures, to using the counting words themselves as countable objects, to counting on from the larger addend after they have developed an informal understanding of the commutative property. Through experiences with and observations of problems set in context, students develop a notion of the commutative property (NRC 2001, 75).

At the same time, children begin using counting down to determine solutions to subtraction problems. There are a number of potential difficulties that arise with this strategy. Baroody (1984) defines counting down as "stating the minuend, counting backward a number of times equal to the subtrahend, and announcing as the answer the last number counted" (205), which is challenging as "it requires two simultaneous processes that in effect go in opposite directions" (205). This is particularly challenging for students who have not mastered counting backward (208). Also, students who struggle with the "number-before-N" idea will greatly struggle here (209), which suggests a necessary first step in curricula if the counting down procedure is to be used. Finally, counting down becomes increasingly more difficult and error prone as the sizes of the subtrahends and minuends increase (206).

Research suggests that we should encourage students to count up, add up, or think of a related addition combination to solve subtraction problems. This is preferable to using counting back because of the difficulties and tendencies toward errors in that method (NRC 2001; Fuson and Kwan 1992). However, research has shown that unless the counting-up procedure is made explicit, students may not invent it until second or third grade, if at all (NRC, 2001). In contrast, if it *is* explicitly taught, it is accessible to first graders and improves accuracy with subtraction to rival addition (191). This is particularly helpful with larger subtrahends and minuends that are not close to each other (Baroody 1984, 207). Also, children may not connect counting up to subtraction, so that connection may need to be made explicit (Baroody 1984, 211).

At this point in their trajectory, students have already made progress toward developing more efficient processes for solving addition and subtraction facts. Much more progress is still to come, though. Baroody (2006) describes three phases students must progress through as they develop mastery with facts: (1) modeling and/or counting to find the answer, (2) deriving answers using reasoning strategies based on known facts, and (3) mastery or efficient production of answers. As students work through the progression, from directly modeling and counting all to the point where they are fluently using recall and strategies, they typically will use multiple approaches simultaneously on different problems and with different numbers throughout their first few years of school; that is, depending on the problem, they may appear to be at different developmental stages (NRC 2001, 188).

Memorization of simple facts such as doubles, adding one, and small totals (sums less than five and their corresponding subtraction facts) begins to occur in late kindergarten and first grade as a by-product of meaningful practice. "Meaningful practice" includes addition and subtraction problems in context, number stories with multiple solutions (e.g. "I have five cars, some are red, some are yellow. How many of each could I have?") and work with ten frames and dot patterns. In contrast, activities designed with the sole and specific goal of encouraging memorization of these facts (e.g., flash cards) are unnecessary and most likely counterproductive to children's mathematical dispositions (NRC 2001, 193).

The ability to derive unknown facts is an important next step leading to computational fluency. Many strategies students apply for deriving facts utilize doubles and combinations of ten (Kling 2011). Doubles are easily memorized (NRC 2001), but this is not necessarily the case with combinations of ten unless they are specifically addressed in the classroom. However, knowing combinations of ten is necessary for developing and using making-ten strategies, which not only promote fluency with basic facts, but further support important strategies in multidigit computation (Barker 2008). Thus learning combinations of ten should be a priority in first grade. This is a reasonable goal; Fuson and Kwan (1992) found that by midway through first grade, most Korean students knew their combinations of ten (161), and that this facilitated their use of making-ten strategies.

Baroody (1985) and Carpenter and Moser (1984) have established that it is not realistic to expect that all families of basic addition or subtraction combinations be mastered in one year. It makes sense, then, to begin focusing on the most critical groups of facts in first grade if we hope that students will have all facts memorized by the end of second grade. Henry and Brown (2008) found that students in their study relied on counting to solve 4 + 6 more so than any other fact in the study, which suggests that familiarity with combinations of ten will not develop on its own without careful, explicit attention to the topic. Finally, Steinberg (1985) found that students who did not know their combinations of ten from recall were less likely to use a making-ten strategy. Thus it is important that combinations of ten are learned early on.

Once students know some facts, they are able to use those facts to derive unknown facts, often using efficient methods and typically beginning in first grade. Steinberg's work (1985) suggests that students need not develop proficiency with counting on before using derived fact strategies. In fact, it may be preferable to introduce some derived fact strategies before students have had time to master counting on, as Steinberg found that students who were very efficient with counting on tended to resist moving away from it.

An important point about strategy development is that "direct teaching of these strategies must be done conceptually rather than simply by using imitation and repetition" (NRC 2001, 188). Furthermore, Steinberg (1985) and Thornton (1978) both used instructional sequences that focused first on doubles, then doubles +/- 1, doubles +/- 2, and combinations of ten. However, Steinberg (1985) found a delay of two to four lessons before students really internalized a strategy. Thus it is important that strategies are not presented in rapid succession if we wish for students to take ownership of them. Furthermore, Thornton's study (1978) strongly indicated that many children seemed to adopt strategies that were explicitly taught, encouraged, or otherwise suggested during instruction. One technique for nudging students toward particular strategies is to focus whole-class discussion around the slightly-more-advanced procedures that some students are using. Those students can be given opportunities to share their procedures, and through discussion and representation, together the class can make sense of why they work. All students can then be encouraged to test out the strategy. To reinforce this work, it is also helpful to discuss the advantages and disadvantages of different strategies (NRC 2001).

For subtraction, similar strategies are used, such as relating to tens, doubles, or related addition facts (fact families). Steinberg (1985) and Thornton (1978) both used an instructional focus of relating subtraction to addition ("think of the addition fact") to teach subtraction. Furthermore, Steinberg stressed using reverse doubles, reverse doubles + 1, and reverse doubles - 1. Also, pointing out the part-part-whole relationships in computation problems, especially subtraction problems, helps to encourage students to think in terms of strategies (NRC 2001, 191).

As children apply, discuss, and practice their addition and subtraction strategies, they begin developing fluency with them. Fluency can be described as "the efficient, appropriate, and flexible application of single-digit calculation skills and is an essential aspect of mathematical proficiency" (Baroody 2006, 22). It often involves children using the relationship of an unknown fact to a nearby memorized fact to efficiently determine the unknown fact without counting. As children practice facts, research suggests that timed testing should be avoided. There are more effective methods of practice that will not "adversely affect students' disposition towards mathematics" (NRC 2001, 193). Henry and Brown (2008) found a negative correlation between timed testing and Basic Facts Competence (fluency) with addition and subtraction facts (169). In contrast, student use of derived-fact strategies in conjunction with retrieval from long-term memory demonstrated a stronger correlation with Number Sense Proficiency than student use of retrieval alone.

Thus for most facts, memorization and instant recall from long-term memory will develop after sufficient "meaningful practice." It does not need to be a product of drill, which actually tends to be less efficient and effective (Baroody 2006, 25; NRC 2001, 193). It seems that practice with thinking strategies is a necessary step toward memorization. Steinberg (1985) found that the only students in her study who were consistently recalling facts by the end of the study were those who had initially used derived facts. This suggests that the use of derived facts is a necessary precursor to fact memorization. Similarly, Carpenter and Moser (1984) found that use of derived facts was often a necessary precursor for children prior to memorizing facts. This is consistent with the trajectory outlined by Baroody (2006), and a key consideration for early elementary curricula.

Multiplication and Division Facts

Children begin developing their concepts of multiplication and division as early as preschool. These concepts are generally based on intuition grounded in grouping or addition; children do not initially understand the binary nature of multiplication as an operation that takes a "set of sets" (Watanabe 2003). Multiplication and division are first learned in the context of word problems, and most children entering kindergarten can use counting to solve simple problems set in context (NRC 2001, 183). Carpenter et al. (1993) confirmed the use of direct modeling as the primary method for solving computation problems (435). As they begin solving multiplication problems, children generally count out the required number of objects for each group needed and then count the total number of objects.

Children typically first solve division problems by either dealing out objects to a predetermined number of groups and then counting the number in each group (partitive) or dealing out a predetermined number of objects to an unknown number of groups and then counting the number of groups (measurement), depending on the context. In either case, as children progress through their first few years of school, Wallace and Gurganus (2005) recommend helping children move from situations that can be modeled using the actual objects in equal groups, to situations that can be modeled with substitute objects using equal groups, to using drawings to represent the equal groups, and finally to representing the groups using tallies. However, a distinguishing characteristic of this level, regardless of the model, is the tendency to apply unitary counting (Mulligan and Mitchelmore 1997; Anghileri 1989).

Research supports the idea of explicitly introducing multiplication in terms of equal groups and arrays beginning in second grade (Mulligan and Mitchelmore 1997; Clark and Kamii 1996; Watanabe 2003; Flowers and Rubenstein 2010; Kouba 1989; Kamii and Anderson 2003). Watanabe (2003) suggests introducing the array model around the same time as equal groups, in second grade. Furthermore, Mulligan and Mitchelmore (1997) found that students had similar success rates on equal groups and array problems. Watanabe (2003) found that Japanese tests include discussions of when multiplication *cannot* be used—the idea of *unequal* groups—in their beginning lessons.

However, several researchers emphasize the importance of distinguishing multiplication from repeated addition; they are conceptually different (Watanabe 2003; Clark and Kamii 1996; Kouba 1989; Mulligan and Mitchelmore 1997). Watanabe (2003) found that U.S. texts often introduce multiplication as repeated addition. Clark and Kamii (1996) echo Piaget's argument that multiplication should not be treated simply as such; multiplication is instead a much more abstract idea than addition. They found that multiplicative thinking—thinking of groups of groups—is clearly distinguishable from additive thinking. Thus, even though multiplication has been formally introduced, we should expect a large percentage of students to not yet grasp the concept of multiplication as a distinct operation in second grade. Clark and Kamii (1996) found that multiplicative thinking was present in only 45% of second graders and develops slowly.

Division can be introduced at the same time by relating it to multiplication. Research suggests that children determine the operation (multiplication or division) by the position of the unknown (Kouba 1989). Thus, if a factor is unknown, children will utilize division, often even if it has not been explicitly taught (Mulligan and Mitchelmore 1997). According to Mulligan and Mitchelmore (1997), children spontaneously relate multiplication and division and do not necessarily find division more difficult. Furthermore, they found throughout their study remarkable improvement on division scores even though division was not explicitly taught. Thornton (1978) also focused on having children "think of the multiplication fact" in her study, leading to improved success on division. Finally, Mulligan and Mitchelmore (1997) found that measurement division seemed to be more difficult for second graders than partitive division, but this resolved itself by third grade.

In second grade, children start applying the idea of repeated addition to solve multiplication problems. This is an advancement over unitary counting; instead now students use rhythmic counting, skip counting, repeated adding, or additive doubling (Mulligan and Mitchelmore 1997). Note there is still no presence of multiplicative thinking; thinking still depends on addition or subtraction principles as opposed to understanding multiplication as a distinct operation. Yet use of repeated addition is still an important milestone. Anghileri (1989) argues that progressing from unitary counting to rhythmic counting in groups is like moving from counting all to counting on strategies for addition.

Although Kouba (1989) found repeated addition to be the main strategy second graders used for multiplication problems, this was not the case for division. Second graders still relied mostly on direct modeling to solve such problems. Children usually start applying repeated subtraction to solve division problems in third grade, which often involves rhythmic counting backward, skip counting backward, repeated subtraction, or additive halving (Mulligan and Mitchelmore 1997). Both Kouba (1989) and Mulligan and Mitchelmore (1997) found that repeated subtraction was used for both measurement and partitive division when children were at this level. Note that there is still no presence of true multiplicative thinking.

However, during third grade, children tend to progress forward one more level, to using repeated addition to solve division problems. Mulligan and Mitchelmore (1997) argue that this is an advancement over repeated subtraction, as children can use this process for both multiplication and division. Additionally, it was found that no children who had progressed to this level used repeated subtraction again. They showed that at the end of third grade, repeated addition was the most common correct strategy used on all division problems except those of a comparison nature. Once again, both Kouba (1989) and Mulligan and Mitchelmore (1997) found that repeated addition was used for both measurement and partitive division when children were at this level.

At this point in the trajectory, students have already made progress towards developing more efficient processes for solving multiplication and division facts. Much more progress is still to come, though. Baroody (2006) describes three phases students must progress through as they develop mastery with facts: (1) modeling and/or counting to find the answer, (2) deriving answers using reasoning strategies based on known facts, and (3) mastery or efficient production of answers. As students work through the progression, from directly modeling and unitary counting to the point where they are fluently using recall and strategies, they typically will use multiple approaches simultaneously on different problems and with different numbers throughout their first few years of school; that is, depending on the problem, they may appear to be at different developmental stages (NRC 2001, 188).

During the first few years of school, through the meaningful practice of skip counting, modeling multiplication and division situations, and working with addition doubles, children begin to memorize the simplest multiplication facts: the 2s, 10s, and 5s (Kamii and Anderson 2003; Heege 1985; Watanabe 2003). Research agrees on the importance of introducing multiplication facts in terms of their relative difficulty, not in terms of factor size as in the traditional progression (Thornton 1978; NRC 2001; Heege 1985; Kamii and Anderson 2003). One discrepancy in the research is the appropriate place for multiplying by 0 and 1. Some research (NRC 2001; Heege 1985) suggests children memorize these easily. However, Watanabe (2003) notes that Japanese texts do not introduce multiplying by 0 and 1 until later (0 not until third grade) because students might not see the need to consider the situation as multiplicative.

As children progress through third grade, they continue to shift towards recognizing multiplication as distinct from addition. Identified as the final stage and called "multiplicative operation" by Mulligan and Mitchelmore (1997), this is the first time students demonstrate explicit understanding of multiplication as a distinct binary operation. At this stage, children are able to either recall facts or derive them using multiplicative thinking primarily. For example, for 6×8, they may say "$5 \times 8 = 40$ and then one more group of 8 makes 48". Note there is no use of direct counting or repeated addition. However, Anghileri (1989) found that the majority of children in her study (75%) used at least three different approaches to multiplication on a single assessment, so these levels are very fluid. She found that even many older children tended to use number patterns rather than recall of multiplication facts.

Children's abilities to recognize multiplication as a distinct operation facilitate the development of strategies based on multiplicative ideas to solve facts of intermediate difficulty. Research shows that the next group of facts, in terms of difficulty to work with and/or memorize, are the squares, 9s, and 3s. Squares are memorized fairly quickly (Thornton 1978; Heege 1985), 3s can be related to doubles (NRC 2001; Flowers and Rubenstein 2010), and 9s can be related to 10s (Thornton 1978). Teachers can engage the class in several routines to help promote this learning of these facts. According to *Adding It Up* (NRC 2001), "finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables" (191). Meaningful practice centered around developing and discussing strategies supports retention and increases performance (NRC 2001, 193). Flowers and Rubenstein (2010) note the value in occasionally having children take inventory of which facts they know versus which they apply a strategy to find, and then observing their progression on the number of facts in each of those two categories as the year continues.

The development of strategies for deriving multiplication and division facts is further promoted as children develop intuitive understandings of the commutative, associative, and distributive properties of multiplication. Children understand and apply these three properties in situations involving multiplication and division, and they form a basis for many derived facts strategies, even if children do not explicitly recognize their use. According to *Adding It Up* (NRC 2001), the commutative property is important for greatly reducing the number of facts to be learned. Heege (1985) identified this as a key strategy used by students. Flowers and Rubenstein (2010) found that rectangles made from grid paper (with factors as dimensions and the product as the area) were good practice for visualizing the commutative and distributive properties of multiplication.

Also, Flowers and Rubenstein (2010) found that the doubling process in multiplication allows the distributive and associative properties to arise naturally. Flowers and Rubenstein identified additional benefits to focusing on doubling in multiplication, including that doubling allows instinctive recognition of decomposing/recomposing numbers and that only the 7s cannot be easily found using doubling, halving, or relating to ten. Heege (1985) argues that memorizing the doubles can support the learning process for multiplication, and identified this as a key strategy. Finally, Thornton (1978) also found doubling to be a key strategy among the fourth graders she studied.

An important point about strategy development is that "direct teaching of these strategies must be done conceptually rather than simply by using imitation and repetition" (NRC 2001, 188). Further, Rathmell (1979) argues for the need to allow sufficient time for strategies to be understood. Steinberg (1985) found a delay of two to four lessons before students internalized a strategy. Thus it is important that strategies are not presented in rapid succession if we wish for students to take ownership of them. Furthermore, Thornton's study (1978) strongly indicated that many children seemed to adopt strategies that were explicitly taught, encouraged, or otherwise suggested during instruction. One way to nudge students toward particular strategies is to focus whole-class discussion around the slightly-more-advanced procedures that some students are using. Those students can be given opportunities to share their procedures, and through discussion and representation, together the class can make sense of why they work. All students can then be encouraged to test out the strategy. To reinforce this work, it is also helpful to discuss the advantages and disadvantages of different strategies (NRC 2001).

As children apply, discuss, and practice their multiplication and division strategies, they begin developing fluency with them. This begins for many children in third grade, and relating to squares (Heege 1985), using doubling, or relating one factor to ten are the most common strategies children use. Fluency can be described as "the efficient, appropriate, and flexible application of single-digit calculation skills and is an essential aspect of mathematical proficiency" (Baroody 2006, 22). It often involves children using the relationship of unknown facts to nearby memorized facts to efficiently determine the unknown without counting or using repeated addition or subtraction. As children practice facts, research suggests that timed testing should be avoided. There are more effective methods of practice that will not “adversely affect students’ disposition towards mathematics” (NRC 2001, 193). Henry and Brown (2008) found a negative correlation between timed testing and Basic Facts Competence (fluency) for addition and subtraction facts (169). In contrast, student use of derived-fact strategies in conjunction with retrieval from long-term memory demonstrated a stronger correlation with Number Sense Proficiency than student use of retrieval alone.

Thus for most facts, memorization and instant recall from long-term memory will develop after sufficient “meaningful practice.” It does not need to be a product of drill, which actually tends to be less efficient and effective (Baroody 2006, 25; NRC 2001, 193). It seems that practice with thinking strategies is a necessary step toward memorization. Dossey and Cook (1982) found more rapid growth in fact mastery when a “thinking strategies” approach is used, and Heege (1985) similarly argues that children become so skilled in applying their informal thinking strategies that they eventually memorize their facts. Similarly, Kamii and Anderson (2003) and Thornton (1978) found that drill was not needed in order to produce high results on a traditional test; engaging and meaningful games and activities was sufficient. This is a key consideration for elementary curricula.

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