This paper provides a detailed summary of the changes made to the fraction content strand from previous editions of *Everyday Mathematics* to *Everyday Mathematics 4*. The recommendations for changes were developed after a careful review of the current research about how children learn mathematics and, more specifically, how they learn about fractions. To read a summary of the research that informed these changes, please see the paper entitled "*Everyday Mathematics 4* Review of Literature for Fractions Strand".

Please note that these recommendations were made prior to the commencement of the formal writing process of *Everyday Mathematics 4*. As Grade Level Leaders worked to enact these, and other recommendations, they often found that they needed to make changes based on what they found as they wrote and field-tested lessons. As a result, you may notice slight differences between these recommendations and the actual content of *Everyday Mathematics 4*. It is reasonable to assume that these differences are the results of decisions made during the formal writing and field-testing process, and were enacted after consulting with the authors of these recommendations. For more information about the writing process or the field testing work for *Everyday Mathematics 4*, please see the "*Everyday Mathematics* and the Field-Testing Process."

Broad-Strokes Changes to the Fraction Strand in *Everyday Mathematics 4*

The recommendations listed in bold below are the broad recommendations that were made for changes across the K-6 curriculum for fractions. Each recommendation is followed by a brief justification for the recommendation.

Develop multiple fraction meanings and lay a foundation for comparison and equivalence through equal partitioning of regions and expanded equal sharing. This requires delaying introduction of standard notation.

The development of fraction concepts in *Everyday Mathematics* Third Edition relies on early introduction of standard fraction notation and subsequently spends substantial time relating this notation to parts of a whole using visuals of collections or pre-divided wholes. Procedures for comparing fractions and finding equivalent fractions based on standard notation are then introduced and frequently used.

This approach of using notation with pre-divided wholes does not allow students to develop deep meanings for fractions during Grades 1–3. It also restricts instruction of fraction comparison and equivalence. It results in an uneven development of meaning and equivalence in Grades 1–3 and a hurried transition to procedures in Grade 4, when operations with fractions are introduced.

In contrast, we recommend an approach that involves first using invented strategies to solve equal sharing problems with fractional answers and to divide shapes into equal parts. Students name the shares and describe their strategies using words and pictures before the introduction of standard notation relating to traditional part-whole representations in Grade 3. This approach allows students to begin with an understanding of fractions as quotients (a single number) and to use their understanding of fair shares to develop meanings for fractions.

Having an understanding of fractions as a single number will help avoid the problem of students applying whole-number reasoning to fractions when they begin to use standard notation – which requires making sense of two numerals (a numerator and denominator) in the same number. For example, this approach can help avoid misconceptions such as comparing fractions by looking just at numerators or denominators or adding both numerators and denominators when adding fractions.

The recommended approach allows for a deeper understanding of fraction concepts in Grades 1 and 2, a smooth transition to notation in Grade 3, and a solid conceptual basis for understanding the procedures for generating equivalent fractions and operations in Grades 4 and beyond.

Use invented strategies to find equivalent fractions and develop methods for fraction comparison and operations.

The approach to ordering and equivalence of fractions in *Everyday Mathematics* Third Edition relies heavily on using fraction cards to compare and find equivalent fractions until students are introduced to formal procedures in Grade 4. Instead of the over-reliance on pre-divided area models, students should use their understanding of fractions as fair shares to invent strategies for solving word problems involving comparison and equivalence. Only after students have come to a deeper understanding of comparison and equivalence of fractions should procedures be introduced.

In *Everyday Mathematics* Third Edition, operations with fractions start in earnest in Grade 4. Because the development of fraction meaning in Grades 1-3 relies heavily on notation, fraction addition is forced to move quickly into notation-based, naked computation problems.

Instead, we recommend that students should begin work with operations on fractions earlier through estimation and invented strategies with problems given in context. This progression more smoothly bridges fraction meaning with the operational procedures, and it is possible because of the changes made to the development of concepts previously discussed. It parallels development of whole number operations in *Everyday Mathematics* through the early use of invented strategies.

Streamline and standardize part-whole representations.

Everyday Mathematics Third Edition uses many different representations when reinforcing concepts and operations with fractions in the later grades. These representations are not consistently used across fractional concepts.

We recommend having a carefully-chosen, consistent set of fraction representations across Grades 3–6. This will allow students to become confident with the same models and deepen understanding by making connections among the representations. Pictures of other models expand students understanding and flexibility with fractions, but maintaining a consistent set of representations across concepts and operations allows students to develop strong mental images of the same models to which they can refer as they compare, estimate, and operate with fractions.

Based on research, we recommend the following set of representations for Grades 3–6:

- *Fraction circle pieces.* These manipulatives serve as a part/whole model that expands fraction concepts developed in earlier grades. Students are then able to use these concepts to develop procedures for operations. Research in classrooms has shown that the fraction circle model is the most effective for building mental images for fractions (RNP, 2009a; RNP, 2009b; Cramer, et al., 2002; Cramer & Wyberg, 2009).[i]
- *Fraction strips (1" x 8.5") for paper folding and developing a fraction chart.* Since students are introduced to this representation through paper folding, it provides a connection to the partitioning activities in Grades 1 and 2. The fraction strips and chart have proven effective for developing mental images that students use for equivalence and comparison (Cramer and Wyberg, 2009). The strips also serve as a transition to number line diagrams.
- *Number line diagrams*. The CCSS call for students to understand fractions as points on the number line in Grade 3. This is a continuation of the idea of a fraction as a single number. Use of number lines brings whole numbers and fractions together in one model, so that students can develop concepts of order and equivalence that include whole numbers. RNP recommends using number lines to deepen understanding of fraction operations by asking student to demonstrate how to add and subtract fractions on number lines after they have used the algorithms (RNP, 2009b).

Summary of Grade-by-Grade Changes to the Fraction Strand in *Everyday Mathematics 4*

The following pages give a broad summary of the changes to the fraction content at each grade level, and are followed by a table that gives more detail about the specific recommendations for the fractions strand at each grade level.

Kindergarten

There are no Common Core State Standards that address fractions in kindergarten, so there is no longer fraction content in kindergarten.

First Grade

The recommended changes to Grade 1 follow the general recommendations for developing fraction meaning through equal partitioning of regions. We recommend deleting use of standard fraction notation in favor of using words to describe partitions.

Second Grade

The recommendations for Grade 2 are similar to those for Grade 1—children partition shapes and share collections instead of counting and naming parts of pre-divided shapes. The purpose of the revision is to avoid misconceptions that arise when children see a fraction as two whole numbers (numerator and denominator) instead of a single number that relates the fractional part to the whole. Children should use invented strategies to solve problems involving partitioning and sharing. They should first describe their results using words.

Third Grade

The changes to Grade 3 are based on efforts to incorporate our reading of the research and to align the content with the CCSS. To that end, our recommendations call for third graders to continue developing meaning for fractions through partitioning and sharing activities and begin to use a consistent set of representations (fraction circle pieces, fractions strips, number line diagrams) to learn to use tools and visual images that will support development of fraction equivalence and comparison. Since use of standard notation for fractions is recommended for deletion from Grade 2, standard notation for unit and non-unit fractions should be introduced in Grade 3.

Fourth Grade

There are significant changes to the G4 fraction strand. Our recommendations call for the removal of all percents and rates from the grade level, as well as removal of probability as a context for fractions. Given the call for a consistent set of representations (fraction circle pieces, fractions strips, number line diagrams) there needs to be a change in the representations for fractions in Grade 4. The fractions work should also entail a gradual development of procedures for addition, subtraction, and multiplication of fractions.

Fifth Grade

The recommendations for Grade 5 assume that the conceptual work with fractions in Grades K-4 will reduce the need for as much emphasis on basic conceptual fraction work in Grade 5. The conceptual work with fractions in Grades K-4 implies that the operations work in Grade 5 can move fairly quickly from invented strategies to procedures. Generally, the operations work should be more contextualized and should consistently refer back to the familiar fraction representations that students have been using.

Sixth Grade

Following the trajectories in the CCSS, the work with fraction concepts and operations is nearly complete by the end of Grade 5. Fraction division is a part of the Grade 6 CCSS, so there should be introduction of, and practice with fraction division – including the standard algorithm of invert and multiply. But the main thrust the fraction work in Grade 6 is to maintain concepts and skills that will support their application in algebra (e.g., solving equations) and extend concepts to ratios and percents. Therefore we recommend a decrease in the number of lessons reviewing fractions concepts and operations, while at the same time recommending revisions that will deepen understanding and maintain skills.

	Meaning	Notation & Representation	Equivalence and Comparison	Operations
	Activities for equal sharing of collections and partitioning regions in which students invent strategies	**Information on representations that support Meaning, Equivalence and Comparison, and Operations.**	**Refer to the Representations column for more information on representations for content by grade.**	**Refer to the Representations column for more information on representations for content by grade.**
K	No fraction standards or work in kindergarten.			
1	• Partition shapes into halves and fourths in multiple ways. • Recognize that more equal shares results in smaller shares • Reassemble the shares to make a whole.	• Represent regions with materials and drawings. • Use words (e.g., *half, halves, half of, one out of two, one-fourth, one out four)* to describe shares and partitions. • Describe the reassembled whole as *two of, three of, or four of* the shares.	• Solve comparison problems in context, e.g., Two girls share a pizza equally. Four boys share the same size pizza equally. Who gets more pizza – one girl or one boy?	
2	• Partition shapes into halves, thirds, and fourths. • Partition shapes in more than one way to show that equal shares do not have to have the same shape.	• Represent regions with paper shapes, student drawings, and pictures. • Use words (e.g., *half, halves, half of, one out of two, one-fourth, two-thirds, one out four)* to describe shares and partitions.		
3	• Partition shapes to build understanding of unit fractions and non-unit fractions. • Share collections of objects in equal shares. • Explore relationships among fraction circle pieces; model and name unit and non-unit fractions using circle pieces. • Represent and name unit and non-unit fractions using paper folding of strips. • Partition and represent unit and non-unit fractions on number lines. • Understand that the whole can	• Represent collections using counters and drawings. • Represent regions using drawings and fraction circle pieces. • Use fractions circle pieces to develop mental images of fractions that support students' understanding as they compare and order fractions. • Use fraction strips as a bridge between part-whole models and number lines. • Introduce standard notation for unit and non-unit fractions. • Express whole numbers as	• Build understanding of fraction equivalence through problems in context using fraction circle pieces, fraction strips, and fraction number lines. • Recognize fractions that are equivalent to whole numbers. • Use problems in context, number lines, and part/whole models to develop strategies to compare fractions.	

	be different sizes and shapes. The size of the fraction depends on the size of the unit whole. • Make connections among fraction representations.	fractions.		
4	• Partition shapes to understand decompositions of fractions and mixed numbers. • Share collections of objects in contexts that result in mixed numbers and fractions. • Model and name fractions and mixed numbers using circle pieces. • Represent and name fractions and mixed numbers using paper folding of strips. • Partition and represent fractions and mixed numbers on number lines. • Construct the unit whole when given a fractional part.	• Represent collections and fractions of collections using counters and drawings. • Represent regions and fraction of regions using drawings and fraction circle pieces. • Represent non-unit fractions as iterations of unit fractions, e.g., ¼ + ¼ + ¼ = ¾. • Use fraction strips to provide a bridge between part-whole model and number lines. • Make connections among fraction representations.	• Solve problems involving equivalent amounts, using both partitioning and discrete models. • Use fraction circle pieces, fraction strips, and number lines to find patterns to identify equivalent fractions and develop a rule for generating equivalent fractions. • Use problems in context to compare and order fractions. • Use fraction circle pieces, fraction strips, and number lines to compare and order fractions.	• Introduce fraction addition through estimation and problems in context where students invent strategies. • Decompose fractions into sums of fractions. • Add and subtract fractions and mixed numbers using fraction circle pieces. • Multiply a whole number by a fraction in problems in context using fraction circle pieces and drawings.
5	• Interpret a fraction as division of a numerator by the denominator (m/p=m÷p).	• Use fraction circle pieces, fraction strips, number lines, and student drawings and representations to support the understanding of fraction operations.	• Relate the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1. • Choose appropriate strategies for comparing fractions to assist in estimating to reason about sums and differences of fractions.	• Extend fraction addition and subtraction using fraction circle pieces to using symbols w/o fraction pieces and to mixed numbers. Estimate sums and differences. Solve problems in context. • Multiply whole numbers, fractions, and mixed numbers using fraction circle pieces and drawings to represent and solve problems in context. • Multiply a fraction x fraction using paper of folding of squares (area model) to develop the algorithm. • Estimate products. Explain why

				multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. Understand multiplication as scaling (resizing). • Divide a unit fraction by a whole number and a whole number by a unit fraction using invented strategies and visual models for problems in context.
6		• Use student drawings and representations, area models, fraction strips, and number lines to solve problems in context for all operations. • Use drawings and number lines to develop algorithms for division.	• Review using appropriate strategies for comparing fractions to assist in estimating for and reasoning about sums and differences of fractions.	• Divide fractions and mixed numbers in context using drawings and number lines. • Explore and apply fraction division algorithms including using reciprocals.

[i] See the "*Everyday Mathematics 4* Review of Literature for Fractions Strand" paper for the full citations.