CENTER FOR ELEMENTARY MATHEMATICS AND SCIENCE EDUCATION
1225 EAST 60TH STREET, CHICAGO, ILLINOIS 60637
P (773) 702-3921 F (773) 702-4312 CEMSE.UCHICAGO.EDU

Everyday Mathematics 4: Foundations for Middle School & Beyond

What prepares students for success in mathematics in middle school and beyond? The EM4 authors believe there are four main drivers: rigor, the mathematical practices, productive dispositions, and a strong technical foundation for algebra.

Rigor

The Common Core defines rigor as equal attention to procedural skill, conceptual understanding, and applications.

Fluency with basics – facts and procedures – is essential for success at higher levels of mathematics. Simple facts must be recalled quickly and basic computational procedures must be carried out automatically in order to free up cognitive resources for higher-level thinking. Learning to multiply polynomials will be almost impossible for students who don't know their multiplication tables.

But knowing basic facts and procedures is not enough: students need to understand why those facts are true and why those procedures work – because without such understandings, the facts and procedures are likely to be mis-remembered or unavailable in situations that call for them. And when students understand not just *how* but also *why*, they can "repair" or "rebuild" procedures and facts that are imperfectly remembered.

But if mathematics is to be useful, then procedural skill and conceptual understanding isn't enough. One must also know how to apply mathematics in realistic situations. This doesn't mean solving a word problem at the end of a chapter by flipping back through the chapter to find a similar, worked example. It means knowing how to apply the mathematics one knows in new situations. This sort of know-how is critical if mathematics is to be useful for solving problems; it's what the Common Core calls *applications*, the third dimension of rigor.

EM4 carefully balances all three dimensions of rigor. In third grade, for example, students learn basic multiplication facts not through rote memorization, but by using strategies, by understanding the properties of multiplication, by connecting fact knowledge with other mathematical ideas, and by applying multiplication in a range of situations, including area, array, and equal grouping problems. By the end of third grade, students begin applying their factual, conceptual, and strategic knowledge to develop methods for multiplying multidigit numbers, which leads in fourth grade to formal algorithms for multi-digit multiplication, which are explicitly connected to area models for multiplication and the distributive property. In fifth and sixth grades, these skills and understandings are extended to the U.S. traditional multiplication algorithm for whole numbers and decimals. Throughout this trajectory, students use what they know to solve a variety of multiplication problems in realistic contexts.

The Mathematical Practices

But mathematics is more than knowledge and know-how. Mathematics is a way of thinking, a set of habits of mind, a culture with norms for behavior. The norms that define the culture of mathematics are what the Common Core calls the mathematical practices. Mastering these practices, becoming enrolled in the culture of mathematics, is essential for long-term success in mathematics.

Becoming socialized in the Common Core's mathematical practices – problem solving, abstraction, reasoning, modeling, and so on – is a long-term endeavor, which is why the Standards for Mathematical Practice (SMPs) are the same across all grades, K-12. But students who have not developed a strong math identity by the end of elementary school are not likely to succeed at higher levels. A key goal of any elementary mathematics program must be to inculcate the mathematical practices in every student.

While the ideas behind the Common Core's mathematical practices have long been central to *Everyday Mathematics*, the EM4 authors have worked hard to make these important and challenging aims accessible to elementary school teachers and students. EM4 breaks down the eight Common Core practice standards into 22 *Everyday Mathematics* Goals for Mathematical Practice (GMPs). EM4 then weaves the SMPs and GMPs into the fabric of regular lessons, Open-Response and Reengagement lessons, and Progress Check lessons.

Productive Dispositions

The Common Core specifies much of what is required for success in middle school and beyond: skills, concepts, applications, and the practices. But the EM4 authors believe another dimension must be attended to as well, something that the National Research Council calls *productive disposition*, the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick, Swafford, & Findell, 2001, p. 5). A belief that what matters for success is effort more than talent – "grit" – is perhaps the single most important productive disposition, but there are other beliefs and attitudes that matter, including curiosity, creativity, and a willingness to take risks. Believing that mathematics is useful and being disposed to enjoy mathematics – experiencing the fun and beauty of mathematics – are also critical. An elementary program that prepares students for success in middle school and beyond must aim at more than just knowledge and practices; it must instill in students a desire to learn mathematics and a belief that they will succeed if they work hard.

While the development of productive dispositions depends heavily on parents, teachers, and the students themselves, the EM4 authors believe a well-designed curriculum can help. Providing students with challenging content in new contexts, as EM4 does regularly in contexts such as the Math Messages and Open Response problems, encourages students to take risks and be creative in attempting problems with solution paths that may be new to them. And EM4's approach, which encourages teachers to foster discussion and student exploration rather than simply showing students how to solve problems, promotes student autonomy and creativity. The enjoyment students find in EM4's many games and other activities is perhaps the best indicator that they are developing dispositions that will serve them well in middle school and beyond.

A Technical Foundation for Algebra

Finally, there are many specific skills and concepts that provide a technical basis for algebra and other topics in higher mathematics. Mastery of arithmetic operations with whole numbers and fractions, for example, provides a template for learning similar operations with algebraic expressions. Understanding partial products multiplication makes learning to multiply polynomials much easier. Understanding the properties of the operations – the commutative, associative, and distributive properties – in arithmetic contexts makes the application of those properties in algebraic contexts seem natural.

Certain of the mathematical practices are also critical to success with algebra. Facility with the Common Core's Standard for Mathematical Practice 2, "Reason abstractly and quantitatively," is especially important. The two big ideas in SMP 2 – being able to represent situations using mathematical symbols and being able to connect mathematical symbols back to the situations they represent – are essential to being able to use algebra to solve problems. The reason for this is that to solve a problem with algebra, the first step is to translate the problem situation into an algebraic equation and the last step is to interpret the solution to that equation in terms of the original problem. These are precisely the two steps students have the most trouble with, and they are also the two big ideas of SMP 2. Similar arguments can be made that facility with SMPs 4, 7, and 8 are critical for success in algebra.

The University of Chicago School Mathematics Project and the EM authors have long recognized the importance of preparing students for algebra. Since its beginnings more than 25 years ago, UCSMP has structured its materials to prepare students for a formal algebra course in eighth grade. Decades of research and development have resulted in EM4 instructional materials that continue and strengthen that tradition.

Conclusion

Building a foundation for success in mathematics in middle school and beyond is a big job. Students need to master basic facts and skills. They must understand the reasons behind those basics and they must be able to use what they know to solve problems. They must learn to think and behave mathematically. And they must master a range of technical skills that form the foundation and template for algebra and beyond. *Everyday Mathematics 4* was designed, built, and tested to help teachers do exactly these things.

Reference

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.