1 *Everyday Mathematics* & the Common Core State Standards

Contents

The University of Chicago School Mathematics Project (UCSMP) aims to improve school mathematics across the entire nation. UCSMP's instructional materials, classroom research, international conferences, and teacher development efforts help teachers better prepare all students for college and career. UCSMP's elementary curriculum, *Everyday Mathematics,* provides the tools elementary school teachers need to meet this long-term goal.

The Common Core also aims to improve school mathematics on a broad scale. The Common Core is at the center of a nationwide effort to prepare all students for college and career, an effort that aims at systemic coherence across instructional materials, teacher development, and assessment.

The goals of *Everyday Mathematics* and the Common Core are thus closely aligned. Both aim at developing all students' mathematical power—their ability to reason, communicate, and solve problems. Both also aim at fostering productive dispositions in students—a belief that mathematics is worthwhile, an inclination to use the mathematics they know to solve problems they face, and confidence in their own mathematical abilities.

For this edition, *Everyday Mathematics* has been rebuilt from the ground up to help teachers teach to the Common Core. The authors coupled their experience developing quality research-based curriculum with a deep understanding of the Common Core's standards in order to unpack them for teachers and students and create useful and effective instructional materials.

The Common Core includes two types of standards: Standards for Mathematical Content and Standards for Mathematical Practice (SMP). The content standards specify procedures, concepts, and applications that students are to master at each grade. The practice standards describe how students should approach the content specified in the content standard. The practice standards are the processes and habits of mind students need to develop as they learn the content standards.

1.1 The Content Standards CCSS 2.NBT.2

The Common Core's content standards are intended to bring greater focus, coherence, and rigor to school mathematics so that students develop deep knowledge of useful mathematics.

1.1.1 Rigor

The Publishers' Criteria, a companion document to the Common Core State Standards, defines rigor as the pursuit, with equal intensity, of conceptual understanding, procedural skill and fluency, and applications (National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2013, p. 3). Defining rigor as balance across skills, concepts, and applications is a significant strength of the Common Core and a great fit with *Everyday Mathematics*.

The assessment consortia, PARCC and SBAC, which are developing high-stakes assessments for the CCSS, use similar language in their frameworks for designing test items. For example, SBAC calls for tasks to assess the rigor of the standards by assessing conceptual understanding, procedural skill and fluency, and applications.

Procedural skill is important for many reasons. Knowing an efficient procedure affords mathematical power, making it possible to solve a whole class of problems with a single method. Fluency with a procedure makes it possible to execute it automatically, which frees up cognitive capacity for higher-level thinking. And knowing a procedure well makes it possible to connect that procedure with other procedures and with related concepts in robust networks of interconnected knowledge that support durable learning and depth of knowledge.

Conceptual understanding is equally important. Students need to understand not only *how* but *why*. Procedural skill without understanding is inflexible and unreliable, limited in scope and utility. Understanding concepts and how they connect to other concepts and procedures is essential to developing deeper understandings and more advanced procedural skills.

This mutual dependence of conceptual understanding and procedural skill has been widely recognized for many decades. What has not been so widely recognized is that applying mathematics is equally important. Much of the mathematics people learn in school languishes unused, gradually fading from memory. Few people leave school appreciating the incredible utility of mathematics for solving problems. Among the many reasons for teaching mathematics, perhaps the most important is its utility, its effectiveness for modeling the world and solving problems. Knowing concepts and procedures is of little value if that knowledge cannot be put to use.

Balancing these three aspects of rigor—procedural skill, conceptual understanding, and applications—has always been fundamental to *Everyday Mathematics*.

1.1.2 Focus

Another key feature of the Common Core is its call for a more focused curriculum, a curriculum that is narrower so that it can be deeper. This narrowing of the curriculum addresses a dilemma in school mathematics: There is too much worth teaching.

Decades of research and development in mathematics education have established that children can learn a great deal of mathematics far earlier than has traditionally been thought. Indeed, the 1989 curriculum and evaluation standards from the National Council of Teachers of Mathematics—the standards that ignited the entire standards movement of the past 25 years—called for a significant broadening of the school mathematics curriculum. Topics such as geometry, statistics, and probability were to be taught at earlier grades than ever before—and the evidence showed that children could indeed learn that mathematics.

There were problems with the new, broader curriculum, however, the most important of which was time. Even with an hour or more of mathematics instruction per day, there is not enough time to teach both traditional topics and the new topics well. Rather than deep and usable knowledge of a broad range of elementary mathematics, students could develop superficial and relatively useless knowledge of that mathematics.

This is the problem that the Common Core's call for focus is meant to solve. By narrowing the range of mathematics to be taught, teachers will have more time to develop deeper, more durable, and more usable knowledge in their students. In the Common Core, for example, there is no longer a rush for mastery of traditional paper-and-pencil algorithms for basic arithmetic operations. The Common Core expectations for those algorithms are later than has been traditional, so that more time can be devoted to students' explorations of diverse strategies based on place value and the principles of the operations. By narrowing the range of mathematics to be taught, more attention can also be devoted to applications and mathematical practices.

This edition of *Everyday Mathematics* achieves focus by adhering closely to the Common Core's content standards. As the Publishers' Criteria notes, focus means focusing on the content standards at each grade (NGA & CCSSO, 2013, p. 3). By any measure, *Everyday Mathematics* achieves such focus. Every activity, every problem, in *Everyday Mathematics* is tightly connected to the content standards, as is shown with the Spiral Tracker, an online tool that provides detailed information about *Everyday Mathematics* and the Common Core. Go Online to Spiral Tracker.

For more information, see Section 2.2.1 *The Spiral: How* Everyday Mathematics *Distributes Learning.*

Section 1

For more information, see Section 2.2 *Design of* Everyday Mathematics.

Note that focus refers to the content to be taught, not how that content is to be taught. For example, a spiraling curriculum such as *Everyday Mathematics* can be intensely focused by returning again and again to key grade-level content through distributed exposures and practice, which has been shown effective by decades of research. Focus does not mean three weeks on one topic, followed by three weeks on another topic, and so on. Research indicates that such an approach is not effective for long-term learning and retention. Nor is such an approach what the Common Core requires.

1.1.3 Coherence

The third key idea in the Common Core's content standards is coherence, the systematic arrangement of content in research-based learning progressions and the weaving together of progressions for different topics in ways that are mutually supportive.

One difficulty with such an approach, of course, is that research-based learning trajectories for all topics in the Common Core do not exist. The logical structure of the mathematics to be learned is clear enough, but the psychological details of how children's learning develops over time are not entirely clear. The Common Core writers and curriculum developers need to fill in gaps in learning trajectories suggested by research and resolve contradictions among different research results. A great deal of professional judgment and design skill is required.

This is an area where field testing and iterative improvement are vitally important. One cannot build an effective learning trajectory without testing it any more than one can build an effective automobile without testing it. The Common Core provides standards and constraints, but only careful engineering work, which the *Everyday Mathematics* authors have done for decades, can yield instructional materials that will work. The careful, research-based, and field-tested arrangement of learning goals and activities has long been a hallmark of *Everyday Mathematics*.

1.1.4 Depth of Knowledge

The Common Core's calls for greater focus, coherence, and rigor in school mathematics all aim at promoting greater depth of knowledge. The idea of depth of knowledge was not invented at the University of Chicago, but the first modern, organized attempt to devise a hierarchy of learning objectives was carried out there by Benjamin Bloom and his colleagues in the 1950s. Bloom's *Taxonomy of Educational Objectives* (1956) organized cognitive behaviors in six levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. In the decades since Bloom's original classification, a number of refinements and alternative formulations have been proposed. Among the most widely cited hierarchy in recent years has been Norman Webb's (1999, 2002) depth of knowledge scheme, which organizes mathematical knowledge into four levels: (1) recall, (2) skill/concept, (3) strategic thinking, and (4) extended thinking. SBAC uses a Cognitive Rigor Matrix that integrates the Bloom and Webb models in their design of assessment items and tasks (Hess, Carlock, Jones, & Walkup, 2009; SBAC, 2012a).

Such hierarchies are founded on the beliefs that some sorts of knowledge are more basic than others and that mastering lower-level knowledge is necessary but not sufficient for mastering higher-level knowledge. The Common Core is well aligned with such thinking. Rigor comprises the full range of content, from facts, procedures, and concepts to applications and non-routine problem solving. Focus, the Common Core's narrowing of the curriculum, is explicitly intended to allow time to reach deeper levels of knowledge. And the coherence of well-articulated learning trajectories is intended to lead students to ever-deeper knowledge of unifying ideas in mathematics.

Everyday Mathematics also aims to go deep—and not only through the focus, coherence, and rigor the Common Core requires. One key advantage of the *Everyday Mathematics* spiral design is that students develop depth of knowledge by repeatedly returning to topics over time, making connections and going deeper with each return. Depth of knowledge is not something that can be developed all at once. Developing deep knowledge requires repeated exposure to key ideas in different contexts and across months or years of time, repeated exposures that a spiral curriculum such as *Everyday Mathematics* is ideally suited to provide.

1.1.5 Unpacking the Content Standards

CCSS 2.NBT.2

The Common Core provides approximately two dozen content standards for each grade, standards that vary widely in grain size and specificity. Some standards are tightly focused on a single concept or skill; others comprise a range of related skills, concepts, and applications. These standards are embedded in a larger framework of clusters and domains that provide context and structure. The Common Core's domains, clusters, and standards provide guidance for the development of curriculum materials and standardized assessments, but do not have the level of detail classroom teachers need for effective instruction, accurate assessment, and targeted differentiation.

Part of the rebuilding of *Everyday Mathematics* has been unpacking the Common Core's content standards into Goals for Mathematical Content (GMC) that are more useful for assessment and differentiation. Consider, for example, the Common Core's first content standard, K.CC.1: "Count to 100 by ones and by tens" (NGA & CCSSO, 2010, p. 11). This simple standard comprises two different goals, counting by ones and counting by tens. In order to be able to assess accurately and differentiate appropriately, *Everyday Mathematics* distinguishes these two goals and tracks each one separately.

Each grade's Common Core content standards are unpacked into 45 to 80 *Everyday Mathematics* Goals for Mathematical Content (GMC). The standards and the corresponding GMCs are listed in the back of each grade's *Teacher's Lesson Guide*. Every instructional item and assessment item in *Everyday Mathematics* is linked to one or more of the GMCs.

The GMCs are called out in various places throughout the program, such as in each lesson's Spiral Snapshot and Assessment Check-In and in every unit's Progress Check lesson's table of content assessed. The digital Spiral Tracker displays complete GMC information about every activity. Go Online to the Spiral Tracker.

For more information, see Section 2.2.1 *The Spiral: How* Everyday Mathematics *Distributes Learning.*

Constructing an intricately structured program such as *Everyday Mathematics* means building fine-grained learning trajectories for the mathematical content specified in the Common Core. Detailed tracking of that content is necessary for accurate assessment and effective differentiation. The GMCs are essential for building such trajectories and carrying out such tracking.

1.1.6 Useful Mathematics

Both *Everyday Mathematics* and the Common Core aim to teach students mathematics they can use—important mathematical ideas with broad implications; logically and psychologically coherent mathematics; mathematics that is balanced across skills, concepts, and applications; mathematics that is powerful.

But teaching students mathematics they *can* use doesn't guarantee that they *will* use it. Students also need to come to *believe* that mathematics is useful. It is not enough that adults believe the mathematics being taught in school is important and useful. The students have to believe it, too. Students also have to come to believe that mathematics is enjoyable and that they are mathematically capable.

The Common Core and *Everyday Mathematics* are designed to develop productive dispositions—habits of mind that will ensure that the mathematics students learn *will* be used. Both aim at producing students who not only know mathematics, but also like mathematics and are disposed to use it to solve problems.

The Common Core's practice standards describe proficiencies that include the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NGA & CCSSO, 2010, p. 6). The following sections examine these Standards for Mathematical Practice.

1.2 Standards for Mathematical Practice CCSS SMP7

The Common Core's Standards for Mathematical Practice (SMP) propose norms for the school mathematics classroom and describe what the classroom culture should be. The eight SMPs are identical across grades K–12, reflecting the expectation that students will develop proficiency with the practices over the course of their school careers. The *Common Core State Standards for Mathematics* (NGA & CCSSO, 2010) provides an overview.

> *The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report* Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).* (p. 6)

The SMPs are a great fit with *Everyday Mathematics*. The SMPs and *Everyday Mathematics* both emphasize reasoning, problem solving, use of multiple representations, mathematical modeling, tool use, communication, and other ways of making sense of mathematics. To help teachers build the SMPs into their everyday instruction and recognize the practices when they emerge in

Everyday Mathematics lessons, the authors have developed Goals for Mathematical Practice (GMP). These goals unpack each SMP, operationalizing each standard in ways that are appropriate for elementary students.

Each practice is addressed below. The "headline" for each Common Core SMP is followed by a list of *Everyday Mathematics* GMPs and a discussion. This section concludes with a vision for classrooms that support and enhance students' development of their own mathematical practices.

Note The Appendix in the *Teacher's Lesson Guide* provides the full text of each of the Common Core's Standards for Mathematical Practice and the corresponding *Everyday Mathematics* Goals for Mathematical Practice. In the Teacher Center, they are also accessible from the Main Menu by clicking on the Grade-Level Resources link. See also the *Teacher's Lesson Guide* ebook.

1.2.1 SMP1 Make sense of problems and persevere in solving them.

GMP1.1 Make sense of your problem.

GMP1.2 Reflect on your thinking as you solve your problem.

GMP1.3 Keep trying when your problem is hard.

GMP1.4 Check whether your answer makes sense.

GMP1.5 Solve problems in more than one way.

GMP1.6 Compare the strategies you and others use.

SMP1 is about problem solving, which has been a focus of recommendations about school mathematics since 1977 when the National Council of Supervisors of Mathematics listed problem solving as the first of its "basic skills." Problem solving has also been the foundation of standards documents from the National Council of Teachers of Mathematics (1989, 1991, 1995, 2000). The emphasis on learning through problem solving in *Everyday Mathematics* is equally longstanding and is fully consistent with SMP1. In the discussions of the other SMPs, you will find that problem solving permeates all the SMPs.

Much of SMP1 is inspired by George Polya's well-known approach to problem solving described in his classic book, *How To Solve It*. Polya outlined four phases in solving a problem: (i) Understanding the problem (ii) Devising a plan (iii) Carrying out the plan (iv) Looking back (1973, pp. xvi–xvii). The GMPs as a whole, and especially GMPs 1.1, 1.4, and 1.6, express Polya's vision of mathematical problem solving and emphasize the importance of students' persistent engagement and ongoing reflection throughout the problem-solving process.

Everyday Mathematics views learning mathematics as a problem-solving activity, so that problem solving cuts across all the content and practice standards. Understanding ideas deeply means connecting them in more and more extensive networks, such as when children connect ideas about sharing to the meaning of division or connect fractions as parts of a whole to fractions as locations or intervals on a number line. Similarly, students are solving problems as they work to explain why their procedures make sense and yield correct results. Students are learning mathematics when they solve problems in more than one way (GMP1.5) and share solution strategies with each other (GMP1.6). As students find success in learning through problem solving, they increasingly reflect on their thinking during the problem-solving process (GMP1.2) and keep trying when the problem is hard (GMP1.3). When problem solving and learning mathematics are merged in a classroom, a student who struggles to make sense of a multistep number story with multidigit numbers may use smaller and easier numbers to represent the problem situation, and then use concrete objects or other representations to show the action (GMPs 1.1, 1.2, 1.3, 1.5). Another student who is grappling with the meaning of a problem may ask questions of peers to help him or her make sense of the situation (GMPs 1.1, 1.3). Such students are demonstrating that they are growing in their proficiency with the SMPs.

1.2.2 SMP2 Reason abstractly and quantitatively.

GMP2.1 Create mathematical representations using numbers, words, pictures, symbols, gestures, tables, graphs, and concrete objects.

GMP2.2 Make sense of the representations you and others use.

GMP2.3 Make connections between representations.

SMP2 is about abstracting, making sense of abstractions, and making connections between abstractions and what they represent. The ability to abstract begins to develop very early. Children see two cats, two cherries, two flowers, and over time understand the meaning of *two*. Later, children understand what *three* is, and so forth. Eventually children come to understand numbers as abstractions that can fit many contexts and even come to understand numbers without reference to any context. When children create mathematical representations by using the numeral 2 or drawing 2 tally marks, they are abstracting from a situation or problem to an abstract representation (GMP2.1). When children connect the numbers or tallies back to the context, for example, "there are two cats in our house," then they are connecting the abstraction back to its original context. They are making sense of the representation (GMP2.2).

2 2 2

Making connections between representations means students are able to move back and forth between different representations of a mathematical situation (GMP2.3). For example, when students first use representations such as base-10 blocks and number grids to subtract multidigit numbers and then connect these representations to symbolic representations of the operation, they are abstracting from the blocks and grids to the symbols. When students make connections between the representations such as explaining how they make trades in a subtraction algorithm by referring to tens and ones as longs and cubes, they show their understanding of place value and the procedure.

SMP2 and Problem Solving

These kinds of connections are critical for problem solving. When students make sense of a problem and are able to represent the mathematics of the situation, they are in a better position to figure out which operations or relationships to use in a solution. When students solve problems in more than one way and compare their strategies to those of other students, the different approaches are often embedded in different representations, so they often need to make sense of various representations.

1.2.3 SMP3 Construct viable arguments and critique the reasoning of others.

GMP3.1 Make mathematical conjectures and arguments.

GMP3.2 Make sense of others' mathematical thinking.

SMP3 is about making claims, or conjectures, about mathematical relationships and regularities and then justifying those claims with arguments based on explicitly stated assumptions, definitions, and previously established results (GMP 3.1).

Mathematical arguments are not the arguments that children often think about—quarrels. Rather, a mathematical argument is logical reasoning used to show why a conjecture or claim is true or false. For example, when making an argument to show that a conjecture is true, a student may use logic to show that the claim works for every possible case in the given domain, such as the whole numbers or the rational numbers. On the other hand, a student can make an argument to show a conjecture is false by showing just one situation in the given domain for which the claim does not hold—this is called a counterexample.

A student might conjecture that in whole numbers, multiples of 10 always have 0 in the ones place. The student's argument could be based on grouping, place value, and the multiplication algorithm. The student could say that a multiple of 10 is any number times 10. Then he or she might say that 10 is 1 ten and 0 ones. Finally, the student could refer to the multiplication algorithm, in which one step is finding the product for the ones place—and the ones place in such a product will always result in a zero, no matter what the non-ten factor is, because any number times 0 has a product of 0. On the other hand, if the conjecture is made that all multiples of 5 have a 5 in the ones place, a student need only point to one counterexample, such as $5 \times 4 = 20$, to show the claim is not true.

Making sense of others' mathematical thinking is essential to the principal work of mathematicians, which is proving and refuting claims and conjectures. Mathematicians submit their conjectures and arguments (or "proofs") to the scrutiny of a community of other mathematicians, just as students do when they pose a conjecture and argument to their peers. When their conjectures and arguments are well articulated, other students can make sense of their mathematical thinking and decide whether the reasoning is correct. When a student's conjecture and its supporting argument are not well articulated, peers ask questions to better understand what is meant, which in turn helps the student construct a clearer conjecture and argument (GMP3.2).

SMP3 and Problem Solving

Constructing viable arguments and critiquing the reasoning of others is essential to mathematical problem solving. When students propose solutions to problems and defend their reasoning, they are beginning to engage in the process of conjecture and argument. In order to make sense of others' mathematical thinking, students must first make sense of the problem and then evaluate the reasoning in the solution and explanation.

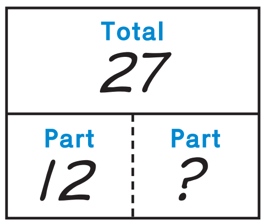
1.2.4 SMP4 Model with mathematics.

GMP4.1 Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations.

GMP4.2 Use mathematical models to solve problems and answer questions.

SMP4 is about mathematizing real-world problems and using the resulting representations to find meaningful solutions. In order to create a mathematical model, students must interpret a real-world situation to identify critical mathematical features that must be included in a mathematical representation, or model, of that situation. The representations could be graphs, drawings, tables, symbols, numbers, or diagrams. (GMP4.1) Students use the model they create to solve the problem, and then they check whether their results are reasonable by mapping the answer back to the real-world situation. If the result is not meaningful, then the student revises the model or generates a new one. The cyclic process continues until a reasonable result is obtained (GMP4.2).

For example, consider this problem: *A total of 27 children are riding on a school bus. 12 of the children are girls. How many children are boys?* A student might represent the situation with a parts-and-total diagram, stripping the situation of the context, while capturing the relationships between the quantities in the problem (GMP4.1).



Parts-and-total diagram

This diagram is a useful model because it clearly reveals the need to find a missing part. No matter what strategy is used to solve the problem (for example, counting up from 12 to 27, or subtracting 12 from 27), an important aspect of the modeling process is to evaluate both the model and answer in the context of the real-world situation. This will show whether the fit between model and situation is reasonable and meaningful (GMP4.2). For example, by asking, "What does *12* mean in this diagram?" the student should be able to connect it back to the context, and know that *12* means *12 girls.*

Modeling with mathematics is closely related to SMP2, reasoning abstractly and quantitatively. SMP2 emphasizes abstraction, including building mathematical representations of various sorts, making sense of those representations, and making connections among representations. SMP4, on the other hand, emphasizes using abstract representations in mathematical modeling and making connections between real-world situations and mathematical representations that model those situations. The mathematical modeling required by SMP4 involves cycles of representing a real-world situation with an abstract model, manipulating the model to produce results, testing the results from the model by mapping them back to the real-world problem, and assessing how well the model worked.

SMP4 and Problem Solving

Modeling with mathematics is problem solving. Students make sense of their problem by mathematizing it, producing an abstract representation that captures the mathematics in the problem situation and omits irrelevant contextual information. When a result is produced from the model, students check to see if that result makes sense in the context of the problem. They may find their model does not produce fully satisfactory results, in which case they need to keep trying, reflecting further on the mathematics of the problem and revising their model to better capture the quantitative relationships in the problem. When students complete multiple cycles of building, testing, and revising a model, they are solving problems in more than one way (usually in improved ways), and comparing the early strategies to those used in subsequent cycles.

1.2.5 SMP5 Use appropriate tools strategically.

GMP5.1 Choose appropriate tools.

GMP5.2 Use tools effectively and make sense of your results.

SMP5 is about students selecting appropriate tools and using them effectively and strategically. Some tools are common across grades (for example, paper and pencil, rulers, calculators, tables, graphs, number lines) and other tools vary by grade level (base-10 blocks, number grids, fraction circle pieces, protractors, **e**Tools). While students may use different tools, one goal of SMP5 is for students to select an appropriate tool for the mathematical work at hand (GMP5.1).

Selecting an appropriate tool is only a first step. One must also use the tool effectively, know its limitations, and identify reasonable and unreasonable results. For example, a student using a calculator effectively to solve a problem will verify that the result is reasonable in case any of the values and/or operations were not keyed in correctly. A student will know the limitations of the tool, for example, recognizing when the remainder to a division problem should be interpreted as a whole number rather than as the decimal value displayed on the calculator. Using tools effectively also includes making strategic decisions. Consider this problem: *Granola bars can be bought in a 3-box pack. Each box weighs 8.9 ounces. What does the 3-box pack weigh?* In this example, whether a student selects a calculator or paper and pencil to solve this problem, multiplying 3×8.9 is more strategic than adding $8.9 + 8.9 + 8.9$.

SMP5 and Problem Solving

Appropriate and effective tool use is critical to problem solving. Students must make sense of a problem before selecting a tool for solving it. They must reflect on their thinking during the solution process in order to decide how to use that tool strategically to solve the problem. Finally, students need to check to see that the answer produced by the tool makes sense.

1.2.6 SMP6 Attend to precision.

GMP6.1 Explain your mathematical thinking clearly and precisely.

GMP6.2 Use an appropriate level of precision for your problem.

GMP6.3 Use clear labels, units, and mathematical language.

GMP6.4 Think about accuracy and efficiency when you count, measure, and calculate.

SMP6 calls for students to keep precision in mind throughout their mathematical work. An important aspect of this SMP is communicating mathematical thinking to others using clear definitions, statements, and representations (GMP6.1). When students try to explain their mathematical thinking to others, they naturally see the need for clearer and more precise mathematical language. The claim that "Mathematics is the universal language" refers in part to the mathematical conventions that exist throughout the world for using labels, units, and mathematical language (GMP6.3). These conventions make discussions among mathematicians and users of mathematics possible across language barriers. For example, everyone knows *by convention* that a square meter means a region bounded by a length of one meter on each side of a square. Although students in *Everyday Mathematics* initially learn concepts and procedures by experiencing them and describing them in their own words, over the years they learn formal terminology and conventional ways of describing quantitative and geometric situations.

A second aspect of SMP6 is using units appropriate for the quantities, sizes, and purposes of the situation (GMP6.2). For example, appropriate units for finding the area of a classroom would be square feet, square meters, or square yards. Each of these conveys a meaningful image of the size of the room to a person who did not make the measurements. Using square miles or kilometers would be inappropriate because the room would measure only a tiny fraction of these units, which would not be helpful in communicating a sense of the size of the room to someone else. Similarly, using square centimeters or square inches would result in such large quantities of each unit that the resulting measure would not be helpful either. The selection of a tool or unit of measure that will minimize error is based on an appropriate level of precision, which depends on the relationships between the quantities in the problem, the units used, and the situation.

A third aspect of SMP6 is calculating and measuring accurately and efficiently. Attending to accuracy is making sure the count, calculation, or measure is close to the true value. Since all measurements are approximations, it is necessary to report them within reasonable measurement error (GMP6.4). For example, if students measuring the length of a line segment *to the nearest half-inch* are doing so accurately, they will report a length of $8\frac{1}{2}$ inches if the measure falls *between* $8\frac{1}{4}$ inches and $8\frac{3}{4}$ inches. Working efficiently is important both in itself and also because it can drive advances in understandings and skills.

SMP6 and Problem Solving

Attending to precision is part of the problem-solving process and an essential component of communicating mathematical thinking and solution strategies to others. Making sense of a problem means you understand the quantities and the relationship among those quantities, so the chosen strategy will result in answers that are appropriately precise. Checking to make sure your answer makes sense is part of ensuring accuracy.

1.2.7 SMP7 Look for and make use of structure.

GMP7.1 Look for mathematical structures such as categories, patterns, and properties.

GMP7.2 Use structures to solve problems and answer questions.

SMP7 is about closely examining a situation to identify a generality in terms of a category, pattern, or property. Searching for and identifying patterns has been part of *Everyday Mathematics* since it was first written. Extending a pattern to a structure means that the description of the pattern is formalized by identifying mathematical categories and properties. For example, first graders solving a series of addition problems ($32 + 10$, $32 + 40$, $32 + 60$) using base-10 blocks will likely represent the 32 as 3 tens and 2 ones, but initially may add 10 ones. The move to adding 40 and 60 encourages children to notice that they could simply add 4 tens (longs) and 6 tens (longs). When students explain why they can use the 4 longs and 6 longs instead of 40 ones and 60 ones, they are noticing a structural property of our base-10 place-value system—that 10 ones is equivalent to the composite unit 1 ten (GMP7.1).

Even more powerful is students' use of mathematical structure to answer questions and solve problems. For example, when learning mental math for multiplication, third graders may use place-value structures when they think of 2×34 as "2 thirties + 2 fours," resulting in a product of $60 + 8$ or 68. Later they learn about the Distributive Property of Multiplication over Addition. They can then reflect back on the pattern they used in mental math. The ultimate goal is for the student to recognize the connection between the mental math strategy and the formal structure, or property, of arithmetic that underlies that strategy: $2 \times (30 + 4) = (2 \times 30) + (2 \times 4)$.

SMP7 and Problem Solving

Finding and using structure is embedded in the problem-solving process. When students reflect on their thinking as they solve a problem, their abilities to apply their knowledge of mathematical structure are enhanced. Further, as students improve their understanding and proficiency in the use of structure, they have the tools to keep trying when solving hard problems.

1.2.8 SMP8 Look for and express regularity in repeated reasoning.

GMP8.1 Create and justify rules, shortcuts, and generalizations.

SMP8 is about using mathematical structures and patterns to create shortcuts or rules that can make procedures and operations more efficient. While SMP7 involves looking for and applying structures and patterns, SMP8 goes further. SMP8 asks students to generalize in order to make solving problems more efficient. Students creating such generalizations and shortcuts need to be able to explain why and how they work so that someone else can use them confidently and with understanding (GMP8.1).

To illustrate, a student may repeatedly use a standard algorithm for subtracting a fraction from a whole number, such as the one below.

$$\begin{array}{r} 3\frac{3}{3} \\ \cancel{4} \\ -\frac{1}{3} \\ \hline 3\frac{2}{3} \end{array}$$

A student who is actively searching for justifiable shortcuts may notice a generalization after solving a few of these types of problems: $2 - \frac{1}{10} = 1\frac{9}{10}$; $1 - \frac{1}{8} = \frac{7}{8}$; and $6 - \frac{1}{6} = 5\frac{5}{6}$. A student may notice that when subtracting a unit fraction from a whole number, the difference is a mixed number made up of the whole number decreased by 1 and a fraction that is one "piece" away from a whole (so the denominator is the same and the numerator is one less than the denominator). A student may justify this shortcut by saying that when you take away a unit fraction from a whole number, you always break up one of the wholes into fractional parts to be able to subtract a unit fraction, so the number of wholes decreases by 1. To subtract just one of the fraction pieces from the whole broken up into parts of the same size, the difference is 1 "piece" less than what is needed to make a whole.

SMP8 and Problem Solving

The proposal and justification of a shortcut, rule, or generalization is similar to conjecture and argument as described in SMP3. The difference is that SMP8 emphasizes the active search for regularity to find efficiencies during problem solving. SMP8, then, is also highly connected to SMP1 because students are continually reflecting on their thinking while solving the problem. Students must also check to see whether a shortcut makes sense by assessing intermediate results during the solution process. Even when a generalization or rule is identified, students continue to search for more efficient ways to solve any repeated calculations.

1.2.9 Standards for Mathematical Practice in the *Everyday Mathematics* Classroom

Teaching Strategies

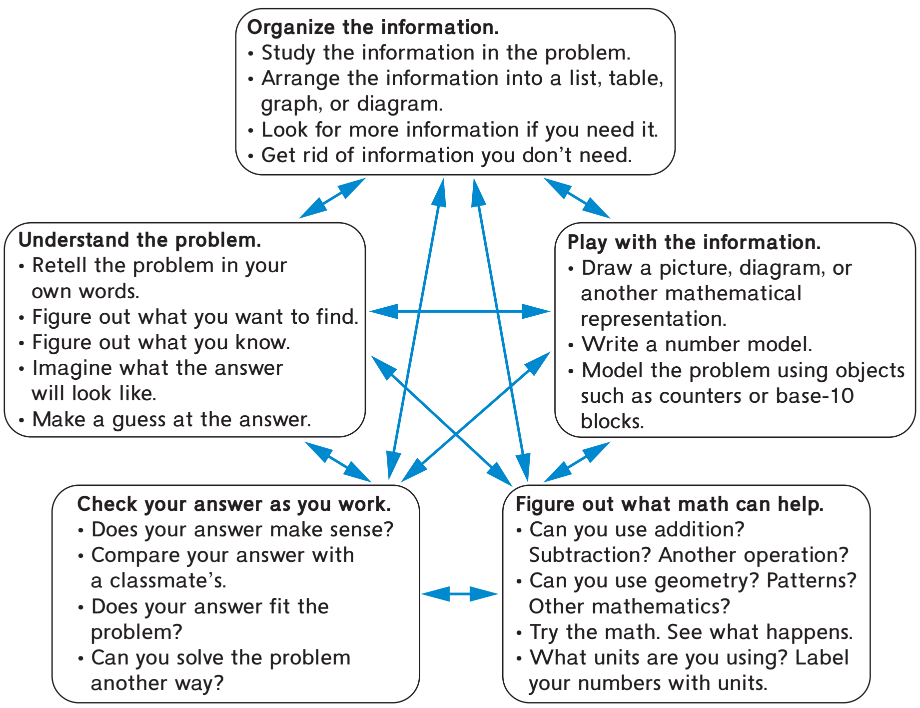
Everyday Mathematics provides specific guidance for teaching mathematical practices targeted in each lesson. The following general strategies are also useful for facilitating students' development of the SMPs:

- Provide opportunities and time for students to grapple with complex problems and analyze other students' strategies and results.
- Maintain high levels of cognitive demand. Do not provide so much scaffolding that students are not challenged to solve problems on their own.
- Encourage students to revise their strategies and results after reflecting on various ways of thinking learned from other students.
- Remind students that struggling to solve complex problems is a natural part of doing mathematics. It can be fun to keep trying when a problem is hard and satisfying when a strategy or solution is improved.

Many students and their teachers feel uncomfortable when students must work hard to mathematically figure out a problem. Alan Schoenfeld's work with college students revealed that even at that level, students felt uncomfortable struggling to solve problems. He found a way to provide help when frustration emerged. To help them reflect on their thinking as they worked on their problems, he asked the following questions: *What are you doing? Why are you doing it? Is it helping?*

Schoenfeld found that over multiple problem-solving experiences, students began to ask these questions of themselves, and they began to learn how to persevere in solving problems. In field tests of the Open Response and Reengagement lessons for this edition, these questions were helpful as they prompted conversation and rethinking among student groups. Sometimes, if frustration continued, groups were asked to split up and observe other groups working on the problem for a few minutes. Then they came back together to discuss what they observed to see whether it would help them solve the problem.

Everyday Mathematics has long used a diagram similar to the one below to convey the dynamic nature of the problem-solving process to students. The diagram captures the spirit of the mathematical practices when thinking about all of the connections and actions across cells and within each cell.



Problem-solving process

The mathematical practices cannot be taught as directly as content is taught. Rather, through their problem-solving experiences and reflections on those experiences, students develop proficiency in the mathematical practices and begin to notice and name those practices. After solving a problem, students can examine their solutions to see how they fit with the targeted GMPs. Student learning of SMPs is a developmental process, so that students' initial ideas are likely to be somewhat crude. *Everyday Mathematics* assumes that while the name of the practice remains the same, students' understanding and ability to articulate the practices will grow over time. The process is

For more information about assessing the mathematical practices, see Section 9 *Assessment in* Everyday Mathematics.

similar to an apprenticeship, in which the apprentice gradually acquires the practices of a trade by engaging in the work of that trade under the watchful eye of a mentor.

SMPs in *Everyday Mathematics* Lessons

The SMPs are addressed in units and lessons the following ways:

- **Highlighted GMPs** Each regular lesson targets one to three GMPs. GMP icons highlight activities and questions that engage students in the targeted practice. Calling out GMPs at point of use gives teachers confidence that they are addressing the practices.
- **Open and Response and Reengagement Lessons** A special two-day lesson is included in each unit and Kindergarten section to provide opportunities for students to engage in targeted mathematical practices as they solve a problem.
- **Writing/Reasoning Prompts** Many Math Boxes have writing/reasoning prompts that encourage students to communicate their understanding of concepts and skills and their strategies for solving problems. Writing/reasoning prompts provide valuable opportunities for engaging in the mathematical practices.
- **Pages in the *Reference Books*** The first section in each *Reference Book* describes the use of each of the practices for students. The pages show children at each grade level solving problems using the SMPs so students can see what is expected when they solve problems and explain their thinking.
- **Standards for Mathematical Practice Posters** These classroom posters list the SMPs and GMPs, providing a daily reminder of these expectations.

For more information, see Section 6 *Open Response and Reengagement.*

For more information on writing/reasoning prompts, see Section 9.3.1 *Assessment Opportunities within Lessons.*

References

Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain.* New York: David McKay.

Hess, K. K., Carlock, D., Jones, B., & Walkup, J. R. (2009). *What exactly do "fewer, clearer, and higher standards" really look like in the classroom? Using a cognitive rigor matrix to analyze curriculum, plan lessons, and implement assessments.* Retrieved from http://www.nciea.org/publications/cognitiverigorpaper_KH12.pdf

National Council of Supervisors of Mathematics. (1977). Position paper on basic skills. Reprinted in *Arithmetic Teacher, 25*(1), 19–22.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2013, April 9). *K–8 publishers' criteria for the common core state standards for mathematics*. Washington, DC: Authors.

Partnership for Assessment of Readiness for College and Careers. (2013). *Principles regarding the common core state standards for mathematics*. Retrieved from http:// http://parcconline.org/mcf/mathematics/principles-regarding-common-core-state-standards-mathematics

Polya, G. (1973). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: MacMillan.

Smarter Balanced Assessment Consortium. (2012a, April 16). *General item specifications*. Retrieved from http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/ItemSpecifications/GeneralItemSpecifications.pdf

Smarter Balanced Assessment Consortium. (2012b, September 7). Building a system to support improved teaching and learning [Webinar]. In *CCSS for Mathematics Webinar Series*. Retrieved from http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/CCSS-for-Mathematic-Presentation-Part-1.pdf

Webb, N. L. (1999). *Alignment of science and mathematics standards and assessments in four states* (Council of Chief State School Officers and National Institute for Science Education Research Monograph No. 18). Madison, WI: University of Wisconsin–Madison, Wisconsin Center for Education Research.

Webb, N. L. (2002, March 28). *Depth-of-knowledge levels for four content areas.* Retrieved from http://facstaff.wcer.wisc.edu/normw/All%20content%20areas%20%20DOK%20levels%2032802.pdf